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JUNE 28, 1915

PREFACE.

THIS book is mainly a *compilation* from other books of Robinson's Series, with some additions of applied examples, drill exercises, etc.

It has been prepared especially for pupils in the third and fourth grades of the Public Schools of the city of Chicago, and in accordance with an outline furnished by the Superintendents of those schools and the Text-book Committee of the Board of Education.

In the preparation of this book care has been taken not to introduce more of theory than is necessary for the illustration and application of the elementary principles of arithmetic, and to compel the pupil at every step to *apply* his knowledge to the affairs of *every day life*, and thus to make it more *practical* than any other similar book before the public.

It has been the aim to go over as many subjects as could appropriately be treated in *third* grade work, and keeping in mind the thorough *oral* training of the preceding grades, much of the simpler matter found in *primary* arithmetics, has been discarded, and the place supplied with condensed, varied, and useful *drill work*.

Facility in the use of numbers is so provided for, by drill and review exercises, as to warrant the assertion, that if the third grade work is well done, according to the plan of this book, the *mechanical* operations of arithmetic will ever after be to the pupil, "an automatic process like the reading of the easiest story-book."

The work for the *fourth* grade is, to a large extent, a comprehensive and enlarged review of the work of the previous grade. Fractions, Decimals, Percentage, and Interest are presented in this grade, but only in their *elements*, "*brief*, and *practical* work," being the motto.

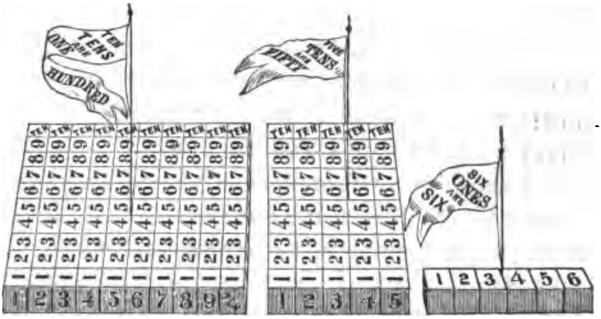
Many pages of *applied* examples have been supplied to break up the monotony of too much *abstract* work, and these examples have been prepared so as to give constant practice on *all previous work*, so that no principle or operation remains long unused.

And while this work has been prepared to meet a particular and local want, it is believed that it will also supply a demand that largely exists in *graded*, and in *city* schools, for an Elementary book, adapted to *intermediate* grades, and which shall *follow* a course of *oral* instruction given in the more primary grades, and be *introductory* to a more advanced and complete treatise on the subject.

It is particularly adapted to that large class of pupils, who do not pass beyond the fourth grade, in our city schools, and to those whose time and opportunities permit them to acquire a knowledge of only the elementary principles of arithmetic, and their application to ordinary business transactions.

August, 1877.

ELEMENTS OF ARITHMETIC.



ONE HUNDRED and FIFTY-SIX. 156.

Art. 1. A *Unit* is a single thing, or a number of single things regarded as a *whole;* as *one, one* ten, *one* boy, *one* year, *one* company.

Any figure standing alone represents one or more *units;* thus, 6 represents 6 *units.*

When *two* figures are written together, the one on the right represents *units,* and the one on the left, ***tens;*** thus, 56 is 5 *tens* or fifty, and 6 *units,* read *fifty-six.*

When *three* figures stand together, the one on the right represents *units,* the next figure, *tens,* and the one on the left, ***hundreds;*** thus, 156 is 1 *hundred,* 5 *tens,* and 6 *units,* read *one hundred fifty-six.*

1. What do the figures 243 represent?

Ans. 2 *hundreds,* 4 *tens,* and 3 *units,* read *two hundred forty-three.*

2. Copy and read the following, naming the *hundreds, tens,* and *units* in each:

341; 473; 184; 537; 208; 782; 960; 872.

2. *1.* In what place are units written? Tens? Hundreds?

2. How many units make one ten?

3. How many tens make one hundred?

4. How many units are 7 tens and 9 units?

5. How many tens are 4 tens and 5 tens?

6. How many hundreds are 3 hundreds and 4 hundreds? How many tens? How many units?

Express in one number, by figures, each of the following:

	Hund.	Tens.	Units.
7. 4 tens, 6 units, and 5 hundreds.	5	4	6
8. 5 units, 8 hundreds, and 4 tens.	8	4	5
9. Six hundreds, eight units, and five tens.	6	5	8
10. Seven tens, no hundreds, and six units.	0	7	6
11. Four units, no tens, and five hundreds?	5	0	4
12. Nine tens, seven hundreds, and no units.	7	9	0
13. Eight hundreds, no tens, and no units.	8	0	0

Write the following numbers in columns, placing units under units, tens under tens, and hundreds under hundreds.

14. 366, 48, 104, 261, 407, 39, and 7.

15. 59, 116, 204, 16, 320, 40, and 10.

16. 408, 37, 100, 9, 750, 20, and 110.

17. What is the greatest number that can be expressed by *one* figure?

18. What is the greatest number that can be expressed by *two* figures?

19. The greatest number that can be expressed by *three* figures is 999.

20. 999 and 1 more equal ***One Thousand, 1000.***

3. *1.* What do the figures 1040 represent?

Ans. 1 thousand, *no* hundreds, 4 tens, and *no* units; read *one thousand forty.*

2. In the same manner copy and read the following:

2406 4051 3007 1904
1572 5200 3333 6070

Write in figures the following numbers:

	Thous.	Hund.	Tens.	Units.
3. Three thousand five hundred seven.	3	5	0	7
4. One thousand two hundred ten.	1	2	1	0
5. Two thousand one hundred three.	2	1	0	3
6. Four thousand thirty.	4	0	3	0
7. Five thousand forty-six.	5	0	4	6
8. Seven hundred eight.		7	0	8
9. Five hundred ninety.		5	9	0
10. Seven thousand seven.	7	0	0	7
11. Six thousand fifty.	6	0	5	0

12. The greatest number that can be expressed by four figures is 9999.

13. 9999 and 1 more equal ***ten thousand.*** 10000

14. What does 1 with *four* 0000's at the right represent? In like manner are represented,

15. 2 ten-thousands, or twenty thousand, by 20000.

16. 3 ten-thousands, or thirty thousand, by 30000.

17. 9 ten-thousands, or ninety thousand, by 90000, etc.

18. 10 units make 1 ten.
19. 10 tens " 1 hundred.
20. 10 hundreds " 1 thousand.
21. 10 thousands " 1 ten-thousand.
22. 10 ten-thousands " 1 ***hundred thousand.***

100000

4. It has been shown that the same figure has a different *value*, according to its place from the right; thus, 6 is 6 *units*, 60 is 6 *tens*, 600 is 6 *hundreds*, etc.

The different places are sometimes called ***orders*** of units; thus, 324 represents 4 units of the *first* order, 2 units of the *second* order, or 2 tens, and 3 units of the *third* order, or 3 hundreds.

The different orders of units are grouped into ***periods*** of 3 figures each.

The first group on the right is called the *period of units*, the second, the *period of thousands*, the third, the *period of millions*, as shown in the following

TABLE.

Periods.	3d.			2d.			1st.		
Name.	Mill.			Thous.			Units.		
Orders of Units.	Hund.	Tens	Units	Hund.	Tens	Units	Hund.	Tens	Units
Number.	1	2	7,	3	6	4,	5	4	9

This number is read one hundred twenty-seven *million*, three hundred sixty-four *thousand*, five hundred forty-nine.

Each *period* is read like a number of *three* figures, giving it the *name* of the period; thus, 120120120 is read, 120 *million* 120 *thousand* 120.

1. In *reading* numbers the name of the units period is omitted.
2. *Every* period except the highest must contain *three* figures.

5. To assist the pupil in learning to write and to read numbers readily, he may be required to prepare on slate or paper, or on the blackboard, exercises similar to the following:

Mill.			Thou.			Units.		
H.	T.	U.	H.	T.	U.	H.	T.	U.
						3	0	7
					2	4	6	0
				1	4	0	8	4
			2	0	5	6	0	2
		2	0	4	0	7	0	0
	3	6	4	0	0	1	4	0

1. The first number is read, 3 hundred 7.

2. The second is read, 2 thousand 4 hundred 60.

3. The third is read, 14 thousand 84.

4. The fourth is read, 205 thousand 6 hundred 2.

5. The 5th is read, 2 million 40 thousand 7 hundred.

6. The 6th is read, thirty-six million four hundred thousand one hundred forty.

After proficiency in smaller numbers has been attained, this exercise may be extended to higher periods.

Copy, point off into periods, and read the following:

7. 3472; 5060; 17043; 20304; 600317; 108300.

8. 500037; 2405037; 910307; 76301: 30406.

9. 2080568; 810096; 40036820; 120620008.

10. Write the preceding numbers in columns, placing units under units, tens under tens, etc.

Write in figures, arrange in columns, point off and read:

11. Nine thousand five hundred twelve; twenty-two thousand nine hundred forty; sixty thousand four hundred eight; ten thousand one hundred fifteen.

12. One hundred twenty-five thousand three hundred eleven; three hundred seven thousand five hundred four; five hundred eleven thousand ten.

ROMAN NOTATION.

6. *1.* This method employs seven capital letters to represent numbers.

LETTERS.	I,	V,	X,	L,	C,	D,	M.
VALUES.	1,	5,	10,	50,	100,	500,	1000.

2. Repeating a letter repeats its value.

Thus, XX represents 20; CCC, 300; DD, 1000.

3. When a letter is placed *after* one of greater value, its value is to be *added* to that of the greater.

Thus, VI represents 6; XV, 15; LXX, 70; DC, 600.

4. When a letter is placed *before* one of greater value, its value is *taken from* that of the greater.

Thus, IV represents 4; IX, 9; XL, 40.

5. When a letter of any value is placed *between* two letters, each of greater value, its value is to be taken from the *united value* of the other two.

Thus, XIV represents 14; XIX, 19.

TABLE OF ROMAN NOTATION.

I = 1
II = 2
III = 3
IV = 4
V = 5
VI = 6
VII = 7
VIII = 8
IX = 9
X = 10
XI = 11
XII = 12
XIII = 13

XIV = 14
XV = 15
XVI = 16
XVII = 17
XVIII = 18
XIX = 19
XX = 20
XXI = 21
XXV = 25
XXX = 30
XXXIV = 34
XL = 40
L = 50

LX = 60
LXX = 70
LXXX = 80
XC = 90
C = 100
CXIX = 119
CC = 200
CCX = 210
D = 500
DCV = 605
M = 1000
MDL = 1550
MDCLXVI = 1666

MDCCCLXXVIII = 1878, one thousand eight hundred seventy-eight.

ADDITION.

7. Counting two or more numbers of the same kind together, to find what number they all equal, is called *adding*, or ***Addition.***

This *sign*, +, is the ***Sign of Addition.*** It is read *plus*, and signifies *more;* thus, 5 + 6 is read, 5 *plus* 6, and means that 5 and 6 are to be *added.*

This *sign*, =, is the ***Sign of Equality.*** It is read *equals*, or *equal to;* thus, 5 + 6 = 11, is read 5 *plus* 6 *equals* 11. It may be read 5 *and* 6 *are* 11.

The number obtained by adding is called the ***Sum.*** Thus, 9 is the *sum* of 5 and 4, or 5 + 4.

Usually, when numbers are to be *added*, they are written in a *vertical* line, or column.

1. I paid 7 dollars for a hat and 9 dollars for a vest; how many dollars did I pay for both?

ANALYSIS.—I paid the *sum* of 7 dollars and 9 dollars, which is 16 dollars.

2. A farmer sold a sheep for 6 dollars and a calf for 8 dollars; what did he receive for both?

3. A boy had 16 marbles and his mother gave him 5 more; how many marbles had he then?

4. James paid 6 cents for a pencil, 5 cents for an orange, and 8 cents for a ball; what did he pay for all?

Find the sum of

5. 9, 7, and 2.
6. 5, 8, and 4.
7. 8, 5, and 7.
8. 15, 5, and 8.
9. 9, 10, and 6.
10. 8, 9, and 10.

EXERCISES FOR THE SLATE AND BOARD.

8. *1.* What is the sum of 34 and 53?

Numbers to be added, { 3 4 = 3 tens 4 units.
5 3 = 5 tens 3 units.
Sum, 8 7 = 8 tens 7 units

Each of these numbers is made up of tens and units. Adding each, the sum is 8 tens and 7 units, or 87 units.

In similar manner add the following:

(2.)	*(3.)*	*(4.)*	*(5.)*	*(6.)*	*(7.)*	*(8.)*
23	44	52	17	26	74	18
55	32	27	71	52	23	60

(9.)	*(10.)*	*(11.)*	*(12.)*	*(13.)*	*(14.)*	*(15.)*
52	34	26	57	74	56	64
23	15	41	31	22	23	14

16. What is the sum of 324, 213, and 431?

Numbers to be added, { 3 2 4 = 3 hunds. 2 tens 4 units.
2 1 3 = 2 hunds. 1 ten 3 units.
4 3 1 = 4 hunds. 3 tens 1 unit.
Sum, 9 6 8 = 9 hunds. 6 tens 8 units.

Each of these numbers is made up of hundreds, tens, and units.

Adding each, the sum is 9 hundreds 6 tens 8 units, or 968 units.

In a similar manner add the following:

(17.)	*(18.)*	*(19.)*	*(20.)*	*(21.)*
213	304	503	104	504
425	123	172	302	670
160	562	223	493	125

(22.)	*(23.)*	*(24.)*	*(25.)*	*(26.)*	*(27.)*
235	112	421	304	142	170
140	231	205	132	210	218
211	354	162	261	134	201

EXERCISES FOR THE SLATE AND BOARD.

9. Copy, add, and give the sum of *each* column:

(*1.*)			(*2.*)			(*3.*)			(*4.*)		
3	3	3	3	3	3	3	3	3	4	4	4
3	3	3	3	3	3	3	3	3	4	4	4
3	3	3	3	3	3	3	3	3	4	4	4
3	3	3	3	3	3	3	3	3	4	4	4
3	2	1	6	5	4	9	8	7	3	2	1
(*5.*)			(*6.*)			(*7.*)			(*8.*)		
4	4	4	4	4	4	5	5	5	5	5	5
4	4	4	4	4	4	5	5	5	5	5	5
4	4	4	4	4	4	5	5	5	5	5	5
4	4	4	4	4	4	5	5	5	5	5	5
6	5	4	9	8	7	3	2	1	6	5	4
(*9.*)			(*10.*)			(*11.*)			(*12.*)		
6	6	6	6	6	6	7	7	7	7	7	7
6	6	6	6	6	6	7	7	7	7	7	7
6	6	6	6	6	6	7	7	7	7	7	7
6	6	6	6	6	6	7	7	7	7	7	7
3	2	1	6	5	4	3	2	1	7	6	5
(*13.*)			(*14.*)			(*15.*)			(*16.*)		
8	8	8	8	8	8	9	9	9	9	9	9
8	8	8	8	8	8	9	9	9	9	9	9
8	8	8	8	8	8	9	9	9	9	9	9
8	8	8	8	8	8	9	9	9	9	9	9
3	2	1	6	5	4	4	3	2	7	6	5

This exercise may at first be written on the board and counted at sight, increasing length of columns, and then omitted entirely.

DRILL EXERCISES.

10. *1.* How many are

$2+2$, $12+2$, $22+2$, $32+2$, etc., to $92+2$?

2. $3+2$, $13+2$, $23+2$, $33+2$, etc., to $93+2$?

3. In like manner add all the different series to $99+2$.

4. Do the same with each of the numbers 3, 4, 5, 6, 7, 8, and 9.

5. Name all the numbers in combinations of two each that make 4. $3+1=4$, $2+2=4$, $1+3=4$.

6. In like manner the combinations that make 5, 6, 7, etc., to 24.

7. Count by 2's from 0 to 100. From 1 to 99.

8. Count by 3's from 0 to 99. From 1 to 100. From 2 to 98.

9. In like manner by 4's, 5's, etc., to 11's.

10. *2, 5, 7, 9, 6, 1, 4, 3, 8.*

Write on the board as above, lines variously arranged, and practice giving promptly at *sight* the sum of each *two* of them; then of *three* at a time; thus, 2, 7, 14; 5, 12, 21, etc. In the same manner until the sum of all is obtained.

11. Count alternately by 2's and 3 s to 35.

Written—2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3.
Counted— 5, 7, 10, 12, 15, 17, 20, 22, 25, 27, 30, 32, 35.

12. Count by 3's and 4's to 49. By 3's and 5's to 48, etc.

These exercises may be written, but should also be counted without being written.

These and subsequent *drill exercises* are designed for *daily practice* in numbers. The exercise should not be made wearisome, *not more than five minutes* being given to it each day.

11. *1.* How many tens in 34 units?

Ans. 3 tens and 4 units.

2. How many tens in 37 units? In 56 units?

3. How many hundreds in 30 tens? In 50 tens?

4. How many hundreds in 36 tens?

EXERCISES FOR THE SLATE AND BOARD.

1. What is the sum of 524, 345, and 473?

524
345
473
Sum 1342

Write units of the same order in the same column.

Begin at the bottom of the units column, and add each column separately, and instead of saying 3 units and 5 units are 8 units, and 4 units are 12 units, name the successive *results* only; thus, 3, 8, 12, the sum of the *units*, equal to 1 ten and 2 units. Write the 2 units in units' place, and add the 1 ten to the lower number in the tens' column; then, 1, 8, 12, 14, the sum of the *tens*, equal to 1 hundred and 4 tens. Write the 4 tens in tens' place, and add the 1 hundred to the hundreds' column; then, 1, 5, 8, 13, the sum of the *hundreds*, equal to 1 thousand and 3 hundreds, which write in the hundreds' and thousands' places.

In the same manner copy, and add:

(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)	(*7.*)
423	304	210	514	75	716
542	718	634	301	610	84
365	532	184	198	393	205

(*8.*)	(*9.*)	(*10.*)	(*11.*)
134 men.	384 boys.	300 pounds.	2036 days
250 "	92 "	480 "	462 "
675 "	807 "	78 "	84 "

12. What is the sum of 203, 176, and 510?

13. What is the sum of 124, 403, and 271?

EXERCISES FOR THE SLATE AND BOARD.

12. *1.* How many bushels of apples in 3 piles, the first containing 26 bushels, the second 35 bushels, and the third 29 bushels?

2. A poultry dealer sold some turkeys for 46 dollars, some geese for 23 dollars, and some chickens for 27 dollars; how many dollars did he receive for all?

3. A farmer sold 38 bushels of wheat, 26 bushels of corn, 18 bushels of rye, and 16 bushels of barley; how many bushels of grain did he sell?

4. In 3 days, Albert picked 23 quarts of strawberries, Harry 29 quarts, Alfred 34 quarts, and Walter 10 quarts; how many quarts did the 4 boys pick?

5. How many pigeons are twenty-eight pigeons, thirty-four pigeons, and twenty-nine pigeons?

6. Frank rode twenty-nine miles Monday, thirty Tuesday, and eighteen Wednesday; how many miles did he ride during the three days?

7. Willie had 47 peaches in his basket, Frederick 16 in his, and Henry 28 in his; how many peaches had the three boys?

8. In one bin there were 121 bushels of wheat, 232 in another, and 167 in another; how many bushels in all?

9. What is the sum of the numbers represented on the face of a clock?

10. What is the sum of 76 feet, 228 feet, and 450 feet?

11. A man paid 375 dollars for a span of horses, 160 dollars for a carriage, and 87 dollars for a harness; what *did he pay* for all?

12. If I were born in 1840, when will I be 63 years old?

13. By selling a farm for 3586 dollars, 684 dollars were lost; how much did the farm cost?

14. A merchant bought at public auction 520 yards of broadcloth, 386 yards of muslin, 92 yards of flannel, and 156 yards of silk; how many yards in all?

15. Five men engage in business as partners, and each puts in 2375 dollars; what is the whole amount of capital invested?

16. What is the sum of 56 miles, 450 miles, and 680 miles?

17. A farmer sold 48 bushels of wheat, 236 bushels of corn, 27 bushels of rye, and 280 bushels of barley; how many bushels of grain did he sell in all?

18. What is the sum of 1942 dollars, and 685 dollars?

19. A woman sold 35 pounds of butter at one time, 42 pounds at another, and 50 pounds at another. How many pounds did she sell in all?

20. A father divided his estate among his four sons, giving each 2087 dollars; what was the amount of his estate?

21. A man engaging in trade, gained 450 dollars the first year, 684 dollars the second, and as much the third as he gained during the first and second; how much was his whole gain?

22. A carpenter built a house for 2464 dollars, a barn for 496 dollars, and fences for 309 dollars; how much did he receive for building all?

23. A has 3240 dollars, B has 5672 dollars, and C has 4050 dollars; how many dollars have all?

ADDITION AT SIGHT.

13. *1.* Any two numbers less than 100, one of which contains only tens, may be added at sight (or orally). Thus, 34+20=54.

Observe, that 3 tens and 2 tens are 5 tens or fifty, and the 4 units make 54.

2. Name the results to the following:

20+ 8
30+12
30+16

40+14
40+26
40+42

50+18
60+33
60+25

80+17
70+27
20+36

3. What is the sum of 23 and 45 ?

ANALYSIS.—23 is equal to 2 tens and 3 units; 45 is equal to 4 tens and 5 units; 2 tens and 4 tens are 6 tens or 60, and 3 units and 5 units are 8 units; 60 and 8 are 68. Hence 23 and 45 are 68.

In like manner find the sum of

4. 12+49.
5. 23+64.
6. 38+17.
7. 51+26.
8. 29+30.
9. 48+34.
10. 72+16.
11. 37+26.

BLACKBOARD DRILL.

14. Place an example upon the blackboard, as the following:

2 6 7 1 8 9
4 8 5 7 3 6
9 1 8 7 5 3
5 9 2 8 4 7
7 0 3 9 2 8
5 6 4 7 8 9

3 5 3 3 2 4 2

Call upon some pupil, who will begin with the right hand column and say, "9, 17, 24, 27, 33, 42 *units*, equal to 4 tens and 2 units; write the 2 units under the column added, and add the 4 tens to the next column." The next pupil will without any delay take up the process, beginning with the 4 tens reserved, and say, "4, 12, 14, 18, 23, 26, 34 *tens*, equal to 3 hundreds and 4 tens; write the 4 tens under the column added and add the 3 hundreds to the next column." So in quick succession let each column be added upwards, then downwards, then from right to left, and from left to right, until the whole class have been exercised *upon this* example.

EXERCISES FOR THE SLATE AND BOARD.

(*1.*) men.	(*2.*) feet.	(*3.*) days.	(*4.*) feet.	(*5.*) miles.	(*6.*) pounds.
542	820	153	1450	2157	1740
176	507	208	1234	1528	2031
628	418	759	2357	1372	1507
473	256	380	1567	2143	1423

(7.)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
1247	5020	1305	3172	1526	4214
2072	1513	6040	1094	5017	2030
4289	3156	3708	7165	2157	5327
3070	1208	1159	2082	1215	1456

Exercise the class on the following, as suggested in Art. **14.**

(*13.*)	(*14.*)	(15.)	(*16.*)	(*17.*)
2046	4250	32607	84065	920684
3678	8786	78982	37987	216677
2766	5968	69771	66789	569911
8345	8789	68339	44321	543344
3875	9896	56234	91389	576677

(*18.*)	(*19.*)	(*20.*)	(*21.*)	(*22.*)
32507	23412	35092	275143	1214187
10325	57638	52803	413100	2742361
47018	15421	47524	650406	1275142
53106	62732	60832	210350	2020560
61007	54298	11462	123415	1603915
27589	17323	74260	113765	1846084

15. Express the answers to some of the following examples, in Roman numerals:

1. How many days in the last four months of the year?

2. What is the sum of all the numbers below 15?

3. How long is your slate? How wide is it?

4. What is the distance around it?

5. Bought two horses; one for 255 dollars and one for 198 dollars. What did they both cost?

6. How old will you be 27 years hence?

7. A man has a 50-cent piece, a 25-cent piece, a 20-cent piece, and a 10-cent piece; how many cents has he?

8. A man was 32 years old when his son was born; how old will he be when his son is 36 years old?

9. The flour in a barrel weighs 196 pounds. If the barrel weighs 21 pounds, what is the weight of flour and barrel?

10. In Illinois there are 102 counties; in Iowa 99, and in Indiana 92. How many counties in the three states?

11. A man traveled 461 miles by railroad, 310 miles by steamboat, and 125 miles by stage. What was the total number of miles traveled?

12. What is the sum of LXVI feet, XLV feet, XCVII feet, XXVIII feet and LXIX feet?

13. From Chicago to Detroit is 284 miles; Detroit to Suspension Bridge, 230 miles; Suspension Bridge to Albany, 304 miles; Albany to New York, 143 miles. What is the distance from Chicago to New York?

14. A merchant deposited 125 dollars in bank on Monday, 91 on Tuesday, 164 on Wednesday, 200 on Thursday, 196 on Friday, and 73 on Saturday; how much did *he* deposit during the week?

SUBTRACTION.

16. The process of finding the *difference* between two numbers of the same kind is called ***Subtraction.***

The greater of the two numbers compared is named the ***Minuend,*** and the less is named the ***Subtrahend.***

The number obtained by subtracting, and which shows how much greater the one number is than the other, is named the ***Difference,*** sometimes the ***Remainder.***

Thus, the *difference* between 12 and 7 is 5, or 5 is the *remainder,* after subtracting 7 from 12.

The *Sign,* –, is the ***Sign of Subtraction.*** It is read *minus,* and signifies *less;* thus, 12 – 7 is read 12 *minus* 7, and means that 7 is to be subtracted from 12.

1. Can you take 6 boys from 11 sheep? Why not?

The difference of two numbers can be found only when they represent things of the *same kind.*

2. Harry bought 12 peaches, and gave five of them to his sister; how many had he left? 12 – 5 = ?

ANALYSIS.—He had left the *difference* between 12 and 5, which is 7.

3. On a tree were 16 pigeons, and 9 flew away; how many were left? 16 – 9 = ?

4. Fourteen books less 5 books are how many books?

5. Asa had 18 marbles, and lost all but 7; how many did he lose?

17. When numbers are so large that the difference cannot be found at once, units may be taken from units, tens from tens, and hundreds from hundreds, etc.

EXERCISES FOR THE SLATE AND BOARD.

1. Subtract 644 from 968.

Minuend,	9 6 8	= 9 hunds.	6 tens	8 units.
Subtrahend,	6 4 4	= 6 "	4 "	4 "
Difference,	3 2 4	3 "	2 "	4 "

PROOF.—Add the difference to the subtrahend. If the work is right, the sum will equal the minuend. Thus, 968 — 644 = 324, the *difference;* and 324 + 644 = 968, the *minuend.*

In like manner subtract and prove the following:

	(2.)	*(3.)*	*(4.)*	*(5.)*	*(6.)*
Minuend,	835	769	578	274	857
Subtrahend,	423	634	453	121	524
Difference,	412				

(7.)	*(8.)*	*(9.)*	*(10.)*	*(11.)*
936	792	478	891	527
214	351	436	170	204

Subtract and prove:

12. 623 from 944.
13. 431 from 862.
14. 354 from 798.
15. 256 from 579.
16. 170 from 382.
17. 322 from 694.
18. 4206 from 9876.
19. 3027 from 7158.
20. 634 from 2865.

21. 899 — 673.
22. 777 — 504.
23. 999 — 657.
24. 886 — 273.
25. 709 — 207.
26. 686 — 475.
27. 8989 — 4736.
28. 1764 — 431.
29. 9628 — 6103.

18. *1.* One ten is how many more than 1 unit?

2. One ten is how many more than 3 units?

3. Two tens are how many more than 7 units?

4. Twenty-five are how many more than 6? than 7?

5. Two tens and 6 units less 8 units are how many?

Analysis.—2 tens and 6 units are 26 units, or 1 ten 16 units; and 8 units from 1 ten 16 units leaves 1 ten 8 units, or 18 units.

6. Three tens 5 units less 1 ten 6 units are how many? 35 — 16 = ?

Analysis.—3 tens 5 units equal 2 tens 15 units; and 1 ten 6 units from 2 tens 15 units leaves 1 ten 9 units, or 19 units.

7. Four tens 6 units less 2 tens 8 units are how many? 46 — 18 = ?

Exercises for the Slate and Board.

19. *1.* Subtract 27 from 84.

Minuend, $\overset{7}{\cancel{8}}\ \overset{14}{\cancel{4}}$
Subtrahend, $\underline{2\ 7}$
Remainder, 5 7

84 is 8 tens and 4 units, 27 is 2 tens and 7 units. Since 7 units cannot be subtracted from 4 units, increase the 4 units by 1 ten or 10 units from the next higher order, making 14 units. 7 units from 14 units leave 7 units, which write in units' place. Since we have taken 1 of the 8 tens, there are only 7 tens left. 2 tens from 7 tens leaves 5 tens, which write in tens' place. Hence, 27 from 84 leaves 57. Proof: 57 + 27 = 84.

Copy, subtract and prove the following:

	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
Minuend,	439	674	523	834	423
Subtrahend,	172	327	247	263	276

(*7.*)	(*8.*)	(*9.*)	(*10.*)	(*11.*)	(*12.*)
4571	5274	7345	9876	6721	2925
2786	1548	5456	4894	3834	1673

20. *1.* A boy had 9 cents, he earned 10 more, then gave 7 to his sister. How many cents had he left?

2. Five pounds, 7 pounds, and 4 pounds are how many less than 20 pounds?

3. Belle had 16 pinks, and gave 3 to Mary, and 5 to Anna. How many had she left?

4. Jennie had 25 cents, and bought some buttons for 8 cents, a pencil for 4 cents, and some thread for 6 cents. How many cents had she left?

5. Ned having 19 cents, lost 4, spent 5, earned 3, and gave away 6. How many cents had he then?

6. How many are 5, 7, and 4, less 2 and 3?

7. How many are 10, 3, and 6, less 5 and 7?

8. How many are 15 less 7, added to 4 and 3?

9. How many are 12 less 5, added to 10 less 3?

10. How many are 13 and 7, less 6 and 9?

11. How many are 10, 7, and 5, less 9 and 3?

12. How many are 14, 5, 9, and 2, less 8 and 7?

13. How many are 25 less 9, added to 18 less 6?

EXERCISES FOR THE SLATE AND BOARD, OR ORALLY.

Copy and write the proper *number* in place of (?):

1. $18 - 7 + 7 = ?$
2. $20 + 18 - ? = 25.$
3. $9 + 11 = 10 + ?$
4. $24 - ? = 7 + 9.$
5. $21 - 9 + ? = 19.$
6. $23 + ? = 12 + 18.$
7. $? + 16 = 26 - 6.$
8. $10 + 13 = ? + 11.$
9. $27 - 7 = 17 + ?$
10. $36 + 4 = ? + 20.$
11. $11 + 7 = 26 - ?$
12. $23 - 5 = ? + 2.$

The teacher should increase the number of examples in every lesson, as the capacity of the class may require, or the time allow.

EXERCISES FOR THE SLATE AND BOARD.

21. *1.* Subtract 279 from 800.

Minuend, $\overset{7}{\cancel{8}}\ \overset{9}{\cancel{0}}\ \overset{10}{\cancel{0}}$
Subtrahend, 2 7 9
Remainder, 5 2 1

Since 9 cannot be taken from 0 units, and since there are *no tens,* we cannot take 1 from that order. Going on to the order of *hundreds*, take 1 hundred, equal to 10 *tens,* leaving 7 hundreds; of these 10 tens take 1 ten or 10 *units*, leaving 9 tens, and the minuend 800 is equal to 7 hundreds, 9 tens and 10 units ($700+90+10=800$). Subtract as before.

In like manner copy, subtract, and prove:

2. 267 from 500.
3. 672 from 740.
4. 504 from 820.
5. 1260 from 3005.
6. 4521 from 6206.
7. 2241 from 7000.
8. 127 from 4300.
9. 32 from 1000.
10. 3030 from 10200.
11. 237 from 8040.

Find the *difference* between

12. 524 and 376.
13. 427 and 806.
14. 900 and 679.
15. 321 and 450.
16. 784 and 508.
17. 907 and 2340.
18. 89 and 1472.
19. 1200 and 164.
20. 2040 and 320.
21. 3672 and 1075.
22. 6804 and 926.
23. 1248 and 3604.

What is the result

24. Of 3416 — 2040.
25. Of 4006 — 844.
26. Of 6400 — 640.
27. Of 2706 — 1371.
28. Of 7120 — 762.
29. Of 5000 — 516.
30. Of 12364 — 1400.
31. Of 8070 — 4105.
32. Of 7346 — 6462.
33. Of 20371 — 8106.
34. Of 16070 — 1284.
35. Of 46137 — 5186.

DRILL EXERCISES.

22. *1.* How many are

10—2, 20—2, 30—2, 40—2, 50—2, etc., to 90—2?

2. 11—2, 21—2, 31—2, 41—2, 51—2, etc., to 91—2?

3. Treat in the same manner all the numbers to 20 — 2; also the numbers 3, 4, 5, 6, 7, 8, and 9, in the place of 2.

4. Name the *parts* of each number from 4 to 24.

Write the number on the board. Let the teacher name one part and the pupils the other. Exercise on each number until all its combinations are used.

5. Count back by 2's from 100 to 0. From 99 to 1.

6. Count back by 3's from 100 to 1. From 99 to 0. From 98 to 2. From 84 to 1.

7. In like manner by 4's, 5's, etc., to 10's.

8. *3, 8, 2, 7, 1, 9, 6, 4, 5.*

Write in a line on the board, as above, numbers variously arranged, and practice giving promptly *at sight* the difference of each *two* of them.

9. Subtract each one in succession from 10. From 11, 12, etc., to 24.

10. 18, +7, —5, —3, +9, +7, —4, +8, —11, +12, —7.

Read, naming results only; thus, 18, 25, 20, 17, 26, etc.

11. 18 — 7, 20 + 9, 23 + 6, 15 — 8.

Place on the board various combinations like the above, and let the pupil make *oral problems* for the same. Thus, for 18 — 7: "I had 18 cents and lost 7 of them. How many had I left?" Another pupil may give the answer and make a problem for the next combination.

The above exercises are for occasional drill.

EXERCISES FOR THE SLATE AND BOARD.

23. (*1.*)

3 4 4
5 7 9
9 2 3
1 8 4 6
3 6 9 2
7 3 8 4

Write the two numbers 344 and 579, and add them without drawing a line, and write their sum 923 as the *third* number; then add the three numbers, and write their sum 1846 as the *fourth* number; then add the four numbers, and write their sum 3692 as the *fifth*, and so on.

The same example may be used for exercise in subtraction, by subtracting from the last result each of the preceding numbers in succession, until *nothing* remains.

In the same manner, copy, add, and prove the following, extending each to the *sixth* number:

(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
327	674	384	540	1257
918	241	609	703	720

EXAMPLES IN ADDITION AND SUBTRACTION.

24. *1.* A man paid 380 dollars for a span of horses, 175 dollars for a carriage, and 90 for a harness. How much did he pay for all? How much more for the horses than the carriage?

2. A lady paid 9 dollars for a bonnet, 45 dollars for a shawl, and had 13 dollars left. How much had she at first?

3. James gave 11 cents for a slate, 26 cents for a book, 5 cents for pens. How much change should he receive for a fifty-cent piece?

4. A man deposited in bank 486 dollars one day, and on the next 327 dollars. How many dollars did he deposit on both days? How much more must he deposit to make the sum 1200 dollars?

5. A railroad company employs 46 engineers, 218 trackmen, 40 conductors, 67 brakemen, and 5 clerks. How many men are employed? How many more trackmen than all others together?

Express by *signs* the solution of the following:

6. How many hours of the forenoon remain at 11 o'clock A. M.?

7. How many hours are past from midnight at 4 o'clock P. M.? How many to midnight again?

8. Queen Victoria was born in 1819; what was her age in 1875?

9. If I sell goods for 387 dollars that cost me 321 dollars, how much is my gain?

10. From DCLXV take CCCXLVII.

11. A man having 625 dollars, receives 75 dollars and pays out 312 dollars. How many dollars has he left?

12. A man had 100 dollars. After paying out a fifty-dollar bill, a twenty-dollar bill, a five-dollar bill, a two-dollar bill, and a one-dollar bill, how many dollars had he left?

13. A grocer had in store 697 boxes of soap, from which he sold in lots as follows: 27 boxes, 75 boxes, 13 boxes, 125 boxes, 37 boxes. How many boxes had he left?

14. How much change, for a dollar, is received when you buy a sheet of music for 25 cents, a quire of paper fo 20 cents, and pencils for 24 cents?

15. $670 - 5 + 40 - 175 + 25 - 215 + 23 - 75 = ?$

16. $18 + 126 - 75 + 51 + 96 - 38 + 200 - 60 = ?$

17. $700 - 99 + 50 + 17 - 360 + 108 - 300 = ?$

18. $1240 - 506 + 100 - 375 + 68 - 250 = ?$

MULTIPLICATION.

25. Taking one of two numbers as many times as there are units in the other is called ***Multiplication.***

The number taken or multiplied is named the ***Multiplicand.***

The number to multiply by, or that shows *how many times* the multiplicand is to be taken, is named the ***Multiplier.***

The result obtained by the multiplication is named the ***Product.***

The *sign,* ×, is the ***Sign of Multiplication.*** It is read *times,* or *multiplied by;* thus, 9 × 7 is read 9 *multiplied by* 7, or 7 *times* 9.

Since changing the order of the multiplicand and multiplier does not change the result, 8 × 6 may be read 6 *times* 8, or 8 *times* 6.

1. If one hat cost 7 dollars, what will 4 hats cost?

ANALYSIS.—Since 1 hat costs 7 dollars, 4 hats will cost 4 times 7 dollars, or 28 dollars.

2. At 5 dollars a cord, what will 6 cords of wood cost?

3. If there are 8 trees in one row, how many trees in 5 rows? In 8 rows? In 9 rows? In 10 rows?

4. If you earn 12 cents an hour, how many cents can you earn in 3 hours? In 5 hours? In 6 hours?

5. How many are 5 times 10 cents, and 8 cents more?

6. How many are 6 times 7 dollars, less 5 dollars?

7. James gave 4 cents apiece for 5 oranges, and had 10 cents left. How much money had he at first?

8. Belle paid 6 cents a yard for 3 yards of ribbon. How much change should she receive for 25 cents?

EXERCISES FOR THE SLATE AND BOARD.

26. *1.* How many are 3 times 42?

BY ADDITION.

Numbers to be added,	4 2
	4 2
	4 2
Sum,	1 2 6

BY MULTIPLICATION.

Multiplicand,	4 2
Multiplier,	3
Product,	1 2 6

The result may be obtained by *addition.* Write 42 *three* times and add; the sum is 126.

But it may be done much shorter and easier by writing the multiplicand 42 but once; and as it is to be taken 3 times, write the multiplier 3 under it in units' place and multiply. Thus, 3 times 2 units are 6 units, which write in units' place; and 3 times 4 tens are 12 tens, or 1 hundred and 2 tens, which write in tens' and hundreds' place. Hence the *product* 126 is the same as the *sum.*

Solve by both methods:

2. 3 times 53.
3. 5 times 61.
4. 4 times 220.
5. 2 times 643.
6. 3 times 4032.
7. 6 times 610.
8. 4 times 3102.
9. 2 times 7234.

	(*10.*)	(*11.*)	(*12.*)	(*13.*)
Multiply	72	136	247	1265
By	4	5	3	6

14. Multiply 2436 by 3; by 5; by 4; by 6.
15. Multiply 3057 by 2; by 3; by 4; by 5; by 6.
16. Multiply 2639 by 3; by 6; by 8; by 4; by 7.
17. Multiply 4244 by 7; by 2; by 5; by 3; by 4.
18. Multiply 3234 by 4; by 6; by 8; by 9.
19. Multiply 2076 by 5; by 6; by 3; by 7; by 4.
20. Multiply 836 by 3; by 7; by 9; by 6; by 5.

MULTIPLICATION TABLE.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	**4**	6	8	10	12	14	16	18	20	22	24
3	6	**9**	12	15	18	21	24	27	30	33	36
4	8	12	**16**	20	24	28	32	36	40	44	48
5	10	15	20	**25**	30	35	40	45	50	55	60
6	12	18	24	30	**36**	42	48	54	60	66	72
7	14	21	28	35	42	**49**	56	63	70	77	84
8	16	24	32	40	48	56	**64**	72	80	88	96
9	18	27	36	45	54	63	72	**81**	90	99	108
10	20	30	40	50	60	70	80	90	**100**	110	120
11	22	33	44	55	66	77	88	99	110	**121**	132
12	24	36	48	60	72	84	96	108	120	132	**144**

27. Let the pupil at sight, or upon hearing a number, name all its sets of *two* numbers, which, multiplied together, will produce it. Thus, 60.

Ans. 6 times 10, or 5 times 12, etc.

4
6
8
9
10
12
14
15
16
20
21
22
24
25
27
28
30
32
35
36
40
42
44
45
48
49
50
55
56
60
63
64
66
70
72
77
81
88
90
96
99
100
108
110
121

Drill Exercises.

28. *1.*

$$\begin{array}{ccccccccc} 7 & 3 & 9 & 1 & 6 & 2 & 4 & 8 & 5 \\ \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} & \underline{2} \end{array}$$

Practice on the above and similar arrangements of the digits, *naming only the products ;* thus, from right to left: 10, 16, 8, 4, 12, 2, 18, 6, 14. Name results *at sight,* not allowing any pause, or time to count fingers.

2. In the same manner use 3, 4, 5, 6, etc., to 12 inclusive, as multipliers.

3. *Double* numbers thus: Take 2. Let teacher say "Twice 2," "Twice 4," "Twice 8," etc., the pupils naming the successive products up to 8192.

4. In the same manner proceed with 3, 5, 7, 9, 11, and 13, keeping the final products below 10000.

5. Using the Multiplication Table on the preceding page, double all the numbers by column, *naming only the results ;* thus, in the first column, 8, 12, 16, 20, 24, 28, etc.

6. Proceed in the same manner with each column of products.

7. In a similar manner practice doubling 20, 200, 2000, 30, 300, 3000, etc.

8. *5, 8, 4, 2, 6, 1, 9, 3, 7.*

Place on the board similar rows of figures variously arranged. Name quickly the product of each *two,* then of each *three.* Multiply the *sum* or *difference* of each *two* by the number following.

9. $4, +3, \times 7, -40, +2, \times 9, +1, -90, -4, \times 7 = ?$

Let the pupils read as follows: 4, 7, 49, 9, 11, 99, 100, 10, 6, 42.

EXERCISES FOR THE SLATE AND BOARD, OR ORALLY

29. Give the result *at sight*, of

3×7 5×3 6×3 9×7 6×7
7×5 8×8 9×9 4×6 8×3
4×9 7×6 8×7 5×8 3×9
9×6 4×7 6×9 7×5 3×10

The pupil may make oral problems for the above. Thus, for "3×7." "What will 7 lemons cost at 3 cents each?" Another pupil may solve the same by a simple analysis; thus, 7 lemons will cost 7 times 3 cents, or 21 cents.

Copy, and write the result in place of (?).

$8 \times 7 + 4 = ?$ $9 \times 8 - 12 = ?$ $7 \times 9 + 10 = ?$
$7 \times 9 + 10 = ?$ $8 \times 6 - 9 = ?$ $8 \times 11 - 8 = ?$
$6 \times 6 + 8 = ?$ $9 \times 9 - 10 = ?$ $7 \times 10 + 12 = ?$
$4 \times 4 + 9 = ?$ $10 \times 8 - 7 = ?$ $11 \times 6 - 9 = ?$
$5 \times 9 + 7 = ?$ $7 \times 11 - 9 = ?$ $8 \times 12 - 11 = ?$

In the following, the numbers given as multiplicands may each be multiplied, separately, by each of the multipliers following it; or first by one of them and the product by the next, and so on.

1. $9704 \times 2, 4, 6, 8.$
2. $658 \times 3, 5, 7, 6.$
3. $1463 \times 4, 2, 3, 5.$
4. $2789 \times 3, 4, 2, 6.$
5. $3596 \times 2, 4, 3, 5.$
6. $2043 \times 4, 5, 2, 7.$
7. $3804 \times 5, 6, 7, 4.$
8. $2964 \times 3, 8, 6.$
9. $4070 \times 5, 9, 4.$
10. $5164 \times 7, 8, 9.$
11. $3005 \times 8, 9, 5.$
12. $10804 \times 6, 7, 8, 9.$
13. $21653 \times 5, 4, 7, 8.$
14. $370250 \times 8, 9, 6, 4, 7.$

MULTIPLICATION AT SIGHT.

30. 8 × 12 = ? 6 × ? = 72 ? × 7 = 84 9 × ? = 81

9 × 6 = ? ? × 5 = 45 12 × ? = 60 ? × 11 = 77

12	?	12	?	7	12	?	?	11
8	8	?	9	?	?	9	12	?
96	96	96						

This exercise may be continued at the option of the teacher.

EXERCISES FOR THE SLATE AND BOARD.

31. *1.* Multiply 362 by 24.

Multiplicand,	362	
Multiplier,	24	
	1448	4 times.
	724	20 times.
Product,	8688	24 times.

Write the multiplicand, and under it the multiplier, the units in units' place, and the tens in tens' place, and multiply by each figure separately.

Multiply 362 by the 4 units, and then by the 2 tens; add the products, and the sum is the entire product, 8688.

In multiplying by the units, write the first figure of the product in units' place. In multiplying by the tens, write the first figure of the product in tens' place.

In the same manner, multiply

2. 487 by 18.
3. 618 by 23.
4. 3241 by 16.
5. 2046 by 25.
6. 894 by 22.
7. 709 by 36.
8. Multiply 30426 by 17; by 28; by 35; by 44; by 42.
9. Multiply 24305 by 52; by 48; by 65; by 72; by 66.
10. Multiply 426173 by 61; by 43; by 53; by 19; by 34
11. Multiply 98583 by 56; by 63; by 64; by 70.

The teacher should increase these examples according to the wants and capacity of the class; also instruct them how to multiply by 10, or 100.

32. *1.* There are 12 inches in a foot. How many inches in 3 feet? In 9 feet? In 12 feet?

2. In 1 square foot there are 144 square inches. How many square inches are there in 14 square feet? In 20 square feet?

3. In 1 cubic foot there are 1728 cubic inches. How many are there in 7 cubic feet? In 12 cubic feet?

4. There are 32 quarts in a bushel. What will a bushel of strawberries cost, at 5 cents a quart? At 10 cents a quart? At 15 cents a quart?

5. A man bought a farm containing 175 acres, at 24 dollars an acre. What did the farm cost him?

6. How many hours in a day? In a week? In a month?

7. What is the sum of the hours in a day, a week, and a month?

8. If the interest on 1 dollar for 1 year is 6 cents, how much is the interest on 25 dollars for the same time? On 39 dollars? On 45 dollars?

9. Bought 5 pounds of sugar at 12 cents a pound, and gave in payment a one-dollar bill. How much change should be returned?

10. Bought 12 yards of cloth at 3 dollars a yard; 6 cords of wood at 5 dollars a cord; and 3 tons of coal at 9 dollars a ton; gave in payment 2 twenty-dollar bills, 5 ten-dollar bills, and a two-dollar bill; how much more ought I to pay?

11. A farmer sold a grocer 8 pounds of butter at 25 cents a pound, and received in payment 15 pounds of sugar at 10 cents a pound. How much was still due the farmer?

Express *answers* to the following examples, in Roman numerals:

12. From Chicago to Omaha is 492 miles. How many miles does an express messenger travel, who goes over the road 4 times in a week?

13. What is the product of the numbers from 4 to 10 inclusive?

14. A farmer having 2150 dollars, bought 536 sheep at 2 dollars a head, and 26 cows at 23 dollars a head; how much money had he left?

15. A has 175 sheep; B has 3 times as many; C has as many as A and B together. How many have they all?

16. A man receives one thousand dollars salary. He pays 364 dollars for board; 275 dollars for clothing; 150 dollars for books, and 125 dollars for other expenses. How much can he save in 7 years?

17. Add LXVII to XLIX; multiply the result by XXXIV, and subtract CXXV.

18. A man bought 2 houses; he paid 2500 dollars for one of them and 1000 dollars for the other. What did he gain by selling both for 4326 dollars?

19. What is the sum of $(504 + 697) \times 4$, and 3000?

20. What is the difference between 19 times 10 less 25, and 17 times 12 less 34?

21. What is the product of $(215 + 82 - 19) \times (108 - 12)$?

22. Which is greater, $28 \times 5 + 7$, or $13 \times (12 + 10)$?

23. How much more is $36 \times (12 + 9)$, than $47 \times (19 - 7)$?

A ***Parenthesis*** ( ) is used to include within it such numbers as are to be considered together as one number.

DIVISION.

33. *1.* At 4 cents each, how many lemons can be bought for 20 cents?

ANALYSIS.—As many lemons can be bought for 20 cents, as 4 cents are contained times in 20 cents, which are 5 times. Hence 5 lemons can be bought for 20 cents.

2. If a man earn 3 dollars a day, in how many days can he earn 12 dollars? 18 dollars? 27 dollars? 30 dollars?

3. If a horse travel 7 miles an hour, in how many hours can he travel 28 miles? 42 miles? 63 miles? 70 miles?

4. If 5 lemons cost 20 cents, what is the price of each?

ANALYSIS.—Since 5 lemons cost 20 cents, 1 lemon will cost 1 *fifth* of 20 cents, or 4 cents.

5. Do you find how many times 5 lemons are contained in 20 cents, or do you find *one* of 5 *equal parts* of 20 cents?

6. How do you find one of 5 equal parts of a number? One of 4 equal parts? One of 7 equal parts?

7. How many times 5 miles are 40 miles?

8. What is one of 5 *equal parts,* or 1 *fifth* of 40 miles?

9. If 18 boys sit on 3 benches, how many boys sit on each bench? 1 *third* of 18 boys are how many boys?

10. If 6 tons of coal cost 36 dollars, what is the cost of 1 ton?

11. If 8 cords of wood cost 40 dollars, what is the cost of 1 cord?

Observe and illustrate that both forms of division are deductions from multiplication. Thus, since 3 times 4 are 12, it follows that 4 is in 12 *three* times, and that 1 *third* of 12 is 4.

34. Finding *how many times* one number is contained in another of the same kind, or finding *one* of the *equal parts* of a number, is called ***Division.***

The number divided is named the ***Dividend.***

The number by which we divide is named the ***Divisor.***

The result obtained by the division is named the ***Quotient.***

The *sign*, ÷, is the ***Sign of Division.*** It is read, *divided by*; thus, 15÷5 is read, 15 *divided by* 5.

When the division is not exact, the part of the dividend remaining is called the ***Remainder,*** and it must always be *less* than the *divisor.*

Thus, in the example, 75 ÷ 8 = 9 and 3 over, 75 is the *dividend,* 8 is the *divisor,* 9 is the *quotient,* and 3, the *remainder.*

Proof: 8 × 9 + 3 = 75.

Exercises for Slate and Board.

35. *1.* Divide 4680 by 2.

Divisor. Dividend.
2) 4680
Quotient, 2340

Write the divisor at the left of the dividend, with a curved line between them, and draw a line under the dividend.' Begin with the highest order and divide, thus: 2 in 4, 2 times; 2 in 6, 3 times; 2 in 8, 4 times; 2 in 0, 0 times, writing each quotient figure under the figure divided, since it is the *same order* of units as that figure. Proof: 2340 × 2 = 4680.

In the same manner, divide and prove:

2. 396 by 3.
3. 5050 by 5.
4. 4884 by 4.
5. 8462 by 2.
6. 9603 by 3.
7. 8008 by 8.
8. 8406 by 6.
9. 5550 by 5.
10. 7007 by 7.

36. *1.* Divide 447 by 3.

Divisor. Dividend. Quotient.
3) 447 (149
3
14
12
27
27
0

In this example, write the divisor at the left, and the quotient at the right of the dividend, and begin at the left to divide, thus: 3 is contained in 4, 1 time and a remainder; write 1 for the first figure of the quotient and multiply the divisor 3 by it, and subtract the product 3 from 4 hundreds, the number divided, and the remainder is 1 hundred, equal to 10 tens, to which add the 4 tens of the dividend, making 14 tens, expressed by bringing down the 4 to the right of the 1 hundred. Then 3 is contained in 14, 4 times and a remainder. Write the 4 in the quotient, multiply the divisor by it, and subtract the product 12 from 14, and the remainder is 2 tens or 20 units, to which add the 7 units of the dividend, making 27. 3 is contained in 27, 9 times. Multiplying and subtracting as before, nothing remains. PROOF: 149 × 3 = 447.

The work may be shortened very much by what is termed *Short Division*, as follows:

3) 447
149

3 is contained in 4, once and 1 remainder; 1 prefixed or placed before 4, makes 14; 3 in 14, 4 times and 2 remainder; 2 prefixed to 7 makes 27; 3 in 27, 9 times. Hence the quotient is 149.

In like manner, divide and prove:

2. 8752 by 4.
3. 7625 by 5.
4. 7122 by 6.
5. 5343 by 3.
6. 8561 by 7.
7. 9024 by 8.
8. 76344 by 6.
9. 90324 by 9.
10. 83210 by 5.
11. 60210 by 6.
12. 57384 by 4.
13. 89005 by 7.
14. 347802 by 3.
15. 987000 by 8.
16. 703218 by 6.
17. 630940 by 5.
18. 927630 by 9.
19. 809263 by 7.

37. *1.* If 6 men earn 24 dollars, what part of 24 dollars does 1 man earn? How many dollars?

2. At 9 cents a quart, how many quarts of milk can be bought for 63 cents?

3. How do the *solutions* of these two examples differ?

4. When the divisor and dividend are of the same name or kind, what do we do? *Ans.* Find *how many times* the dividend contains the divisor.

5. What is the quotient? *Ans. Times.*

6. When the divisor and dividend are not of the same name or kind, what do we do?

Ans. Find a *certain part* of the dividend.

7. What is the quotient? *Ans.* A *part* of the dividend.

8. How many yards of cloth, at 7 dollars a yard, can be bought for 70 dollars? For 63 dollars? For 84 dollars?

9. If you divide 84 cents among 7 children, what part of the whole do you give to each? How many cents?

EXERCISES FOR THE SLATE AND BOARD.

1. How many times is 6 contained in 1834?

Divisor. Dividend.
$$6\,\overline{)\,1834\,}$$
Quotient, $305\frac{4}{6}$

Since 6 is not contained in 1, say, 6 in 18, 3 times and no remainder; 6 in 3, 0 times and 3 remainder; 6 in 34, 5 times and 4 remainder, which write over the divisor 6, as a part of the quotient. PROOF: $305 \times 6 + 4 = 1834$.

In like manner,

2. Divide 3324 by 2; by 4; by 6; by 7; by 8.

3. Divide 9106 by 3; by 4; by 5; by 8.

4. Divide 60530 by 5; by 6; by 7; by 8; by 9.

5. Divide 65625 by 3; by 4; by 7; by 8; by 9.

ANOTHER FORM OF EXPRESSING DIVISION.

38. *1.* Division is sometimes indicated by placing the divisor under the dividend, and separating them by a line.

Thus, $\frac{12}{4}$ means 12 divided by 4, or 1 *fourth* of 12, and is the same as $12 \div 4$, the quotient being 3.

2. Find the quotient, or value, of each of the following:

$\frac{36}{9}$; $\frac{40}{5}$; $\frac{42}{7}$; $\frac{56}{8}$; $\frac{54}{9}$; $\frac{63}{7}$; $\frac{72}{6}$;
$\frac{42}{6}$; $\frac{50}{10}$; $\frac{45}{9}$; $\frac{60}{6}$; $\frac{56}{7}$; $\frac{64}{8}$; $\frac{81}{9}$.

This form of indicating division often simplifies two or more operations that are to be performed; thus,

3. It is required to multiply 12 by 5, and divide the product by 6.

Written, $\frac{12 \times 5}{6} = 10$; for, $12 \times 5 = 60$, and 1 *sixth* of 60, or $60 \div 6 = 10$.

4. To 7 times 8 add 4 and divide the sum by 6.

Written, $\frac{8 \times 7 + 4}{6} = 10$; for, 8×7 is 56, plus 4 is 60, and 1 *sixth* of 60, or $60 \div 6 = 10$.

Express by signs each of the following:

5. Divide the difference between 17 and 5 by 4.

6. Divide the product of 12 and 4 by 8.

7. What is 1 *seventh* of the sum of 34 and 8?

8. From the sum of 25 and 10 subtract 5, and divide the remainder by 6.

9. From the product of 10 and 7 subtract 6, and divide the remainder by 8.

10. Multiply the sum of 12 and 10 by the difference between 15 and 9, and divide the product by 3 times 4.

39. *1.* What is 1 half of 12? Of 16? Of 18+6?

2. What is 1 third of 15? Of 18? Of 21+9?

3. What is 1 fourth of 16? Of 24? Of 30+6?

4. What is 1 fifth of 20? Of 30? Of 40 — 5?

5. What is 1 sixth of 24? Of 36? Of 38+10?

6. What is 1 seventh of 35? Of 42? Of 60 — 11?

7. What is 1 eighth of 48? Of 32? Of 70 — 6?

8. What is 1 ninth of 54? Of 63? Of 60+12?

9. What is 1 tenth of 90? Of 87 — 7? Of 62+8?

10. What is $\frac{1}{3}$ of 4 times 9? $\frac{1}{4}$ of 5 times 8?

11. What is $\frac{1}{7}$ of $60 - 4$? $\frac{1}{6}$ of 12×4? $\frac{1}{8}$ of $66 + 6$?

12. What is $\frac{1}{4}$ of 6×6? $\frac{1}{9}$ of 12×6? $\frac{1}{6}$ of $75 - 9$?

EXERCISES FOR THE SLATE AND BOARD.

40. Find the value of each of the following expressions in a single number:

(*1.*) $\frac{21 - 9}{6}$ (*2.*) $\frac{43 - 7}{9}$ (*3.*) $\frac{12 \times 5}{10}$ (*4.*) $\frac{9 \times 8}{12}$

(*5.*) $\frac{7 \times 9 + 7}{7}$ (*6.*) $\frac{8 \times 9 - 8}{8}$ (*7.*) $\frac{6 \times 9 + 10}{8}$ (*8.*) $\frac{12 \times 7 + 6}{9}$

(*9.*) $\frac{13 + 34}{12}$ (*10.*) $\frac{42 + 36 - 14}{8}$ (*11.*) $\frac{45 \times 4 + 4}{2}$

(*12.*) $\frac{128 \times 3}{4}$ (*13.*) $\frac{28 \times 16}{8}$ (*14.*) $\frac{242 \times 21}{7}$

(*15.*) $\frac{14 \times 9 - 6}{8 + 4}$ (*16.*) $\frac{240 - (27 \times 3)}{41 - 38}$ (*17.*) $\frac{(16 + 24) \times (36 - 14)}{4 \times 2}$

41. *1.* If 4 caps cost 12 dollars, what will 6 caps cost?

SOLUTION.—Since 4 caps cost 12 dollars, 1 cap costs 1 *fourth* of 12 dollars, or 3 dollars; and 6 caps will cost 6 *times* 3 dollars, or 18 dollars.

2. How many dollars are 6 times 1 fourth of 12 dollars?

3. If 6 oranges cost 18 cents, what will 8 oranges cost?

4. If 7 yards of ribbon cost 56 cents, what will 3 yards cost? 5 yards? 8 yards? 6 yards? 9 yards?

5. If a man earn 30 dollars in 6 days, how many dollars does he earn in 4 days? In 3 days? In 5 days?

6. How many yards of cloth can I buy for 36 dollars, if 3 yards cost 12 dollars?

7. If a man walk 27 miles in 8 hours, how far does he walk in 7 hours?

8. How many are 8 times 1 sixth of 42? 9 times 1 fifth of 35? 10 times 1 half of 18? 6 times 1 ninth of 72?

EXERCISES FOR THE SLATE AND BOARD.

1. If 6 acres of land are worth 366 dollars, what are 8 acres worth?

2. If there are 174 lines on 6 pages of this book, how many lines on 16 pages?

3. If 8 cords of wood are worth 32 dollars, what are 27 cords worth?

4. What will 45 pounds of beef cost, if 8 pounds cost 96 cents?

5. If 10 tons of coal are worth 70 dollars, what are 125 tons worth?

6. If a man travel 126 miles in 7 days, how far will he travel in 16 days?

EXERCISES FOR THE SLATE AND BOARD.

42. *1.* Divide 3180 by 15.

```
15)3180(212
   30
   --
    18
    15
    --
     30
     30
     --
```

When the divisor consists of two or more figures, the operations of multiplying and subtracting cannot well be carried on in the mind, and so we write the result of each separate operation.

Since 15 is not contained in 3, we say 15 in 31, 2 times, and write the 2 at the right of the dividend as the first figure of the quotient. Multiply the divisor by this quotient figure, and write the product 30 under the figures divided.

Subtract, and to the remainder 1, annex 8, the next figure of the dividend, making 18 for a new dividend.

Dividing, multiplying, and subtracting as before, we have a remainder of 3, to which annex the 0 of the dividend, and we have a new dividend of 30; which divide as before, and nothing remains.

PROOF: $212 \times 15 = 3180$.

2. Divide 12708 by 28.

Divisor. Dividend. Quotient.

```
28)12708(453 24/28
   112
   ---
    150
    140
    ---
     108
      84
     ---
      24 Remainder.
```

PROOF.

```
  453 Quotient.
   28 Divisor.
 ----
 3624
 906
-----
12684
   24 Remainder.
-----
12708 Dividend.
```

If the *product* of the divisor by the quotient figure is *greater* than the part of the dividend divided, the quotient figure is *too great* and must be *diminished.* If the *remainder* after any subtraction is *greater* than the divisor, the quotient figure is *too small,* and must be *increased.*

3. Divide 23546 by 12; by 14; by 15; by 18.

4. Divide 37985 by 24; by 27; by 35; by 36.

43. *1.* How many times is 5 contained in 10 × 6?

2. How many times is 3 × 3 contained in 63?

3. Divide 48 by 1 sixth of 24. By 1 fourth of 32.

4. A boy gave 5 peaches to each of 6 boys, and had 1 third as many left. How many had he left?

5. Mary has 5 cents and Susie 7. If Willie has 4 times as many as both, how many cents has he?

6. What is 1 fourth of 6 times 8? 1 third of 5 times 6?

7. A man sold 8 barrels of apples at 5 dollars a barrel. How many yards of cloth, at 4 dollars a yard, can he buy with the money?

8. Three boys have some marbles. James has 10, Henry 9, and John 4. If they divide them equally, how many marbles will each have?

9. How many days work, at 4 dollars a day, will pay for 3 tons of coal worth 8 dollars a ton?

Exercises for the Slate and Board.

1. Divide 32760 by 12; by 13; by 14; by 15.

2. Divide 970640 by 23; by 34; by 25; by 36.

3. Divide 40320 by 12; by 24; by 15; by 16.

4. Divide 816480 by 24; by 27; by 35; by 36.

5. Divide 445280 by 32; by 41; by 28; by 33.

6. How many pounds of sugar, at 14 cents a pound, will pay for 13 pounds of butter, at 28 cents a pound?

7. How many pounds of coffee worth 25 cents a pound, can be bought for 15 bushels of oats worth 65 cents a bushel?

8. How many tons of hay at 16 dollars a ton, must be given for 24 cows worth 32 dollars a head?

44. *1.* A boy having 12 oranges bought 6 more, and then sold 7; how many had he left?

2. James sold his sled for 45 cents, which was 9 cents more than it cost; what did it cost?

3. Belle paid 8 cents a spool for 7 spools of thread, and 9 cents for some buttons; what did she pay for both?

4. Which is less, 7 times 8 or 6 times 9? Which is greater, 5 times 12 or 8 times 8?

5. What will be the cost of 7 coats at 12 dollars each?

6. If Jane has 11 cents and Grace has 6 times as many, how many cents has Grace? How many have both?

7. If John has 40 marbles and Dick has 1 fourth as many, how many marbles has Dick?

8. If 6 quarts of milk cost 54 cents, what will be the cost of 4 quarts? 7 quarts? 9 quarts? 8 quarts?

9. Willie sold his knife for 36 cents, and received in payment 3 quarts of chestnuts at 8 cents a quart, and the remainder in money; how much money did he receive?

10. If a man can chop 16 cords of wood in 8 days, how much can he chop in 3 days? In 5 days? In 7 days? In 12 days?

11. How many barrels of flour worth 8 dollars a barrel, will pay for 4 loads of hay worth 12 dollars a load?

12. If 7 yards of cloth cost 28 dollars, what will 5 yards cost?

13. If 1 yard of ribbon cost 10 cents, how many yards can be bought for 60 cents? For 70 cents? For 90 cents? For 120 cents?

14. How many pounds of rice at 6 cents a pound, will cost as much as 8 pounds of meat at 9 cents a pound?

EXERCISES FOR THE SLATE AND BOARD.

45. *1.* The greater of two numbers is 590, and the less 364; what is their difference?

2. The less of two numbers is 128, and their difference is 75; what is the greater?

3. The sum of two numbers is 405, and one of the numbers is 214; what is the other?

4. How many must be added to 24 to make 56?

5. How many more than 124 men are 216 men?

6. How many less than 72 sheep are 27 sheep?

7. How many more than 65 + 36 is 124?

8. How many less than 184 + 153 is 246?

9. How many must be added to 74 + 127 to make 304?

10. A man paid 3146 dollars for a house, which was 380 dollars more than it was worth; what was it worth?

11. From 1705, take the sum of 540 and 603.

12. From the difference of 3242 and 646 take 845.

13. Bought a house for 5607 dollars, which was 825 less than it was worth; what was it worth?

14. A man owed 756 dollars, and paid at one time 206 dollars, and at another time 324 dollars. How much did he still owe?

15. Mr. Smith has 640 acres of land, and Mr. Jones has 124 acres less; how many acres has Mr. Jones?

16. From a bin containing 394 bushels of oats, 248 bushels were taken, and afterwards 86 bushels returned; how many bushels were then in the bin?

17. A house that cost 1500 dollars, and the repairs 315 dollars, was sold for 2000 dollars; what was the gain?

EXERCISES FOR THE SLATE AND BOARD.

46. *1.* If a steamer sail 265 miles a day, how far will she sail in 16 days? In 18 days? In 24 days?

2. A man bought 136 barrels of flour at 8 dollars a barrel, and sold the whole for 1248 dollars. What was his gain?

3. What is the value of each of the following articles: 25 pounds of sugar at 12 cents a pound; 16 pounds of tea at 75 cents a pound; and 38 pounds of ham at 14 cents a pound?

4. What is the value of the whole? How much more is the tea worth than the sugar?

5. A man earns 9 dollars a week, and his son 5 dollars; what will both earn in 3 weeks? In five weeks?

6. How much more will the father earn in 12 weeks than the son?

7. If a clerk earn a salary of 950 dollars a year, and his expenses are 525 dollars a year, how much can he save in 6 years?

8. A farmer sold 96 bushels of potatoes, at 52 cents a bushel. How many pounds of coffee, at 26 cents a pound, will pay for the potatoes?

9. If 24 yards of cloth cost 144 dollars, what will 56 yards cost at the same rate?

10. A farmer sold 15 tons of hay, at 18 dollars a ton, and 36 cords of wood, at 4 dollars a cord; what was the value of both?

11. He divided the whole amount of money equally among 6 creditors: how much did each receive?

47. *1.* Multiply 3417 by 9; by 12; by 14; by 16.
2. Multiply 10782 by 21; by 18; by 26; by 19.
3. Multiply 56043 by 32; by 25; by 36; by 27.
4. Multiply 28340 by 41; by 29; by 46; by 37.
5. Multiply 730081 by 44; by 53; by 38; by 54.
6. Multiply 186304 by 65; by 67; by 55; by 49.
7. Multiply 304071 by 46; by 72; by 84; by 66.
8. Multiply 415036 by 75; by 63; by 54; by 82.
9. Multiply 630400 by 46; by 57; by 68; by 74.
10. Multiply 297182 by 77; by 94; by 95; by 89.
11. Multiply 710345 by 68; by 77; by 81; by 93.
12. Multiply 5423568 by 45; by 73; by 84; by 96.

48. Find the value of

1. 598467÷6.
2. 541604÷8.
3. 639514÷4.
4. 954632÷7.
5. 985478÷7.
6. 517401÷8.
7. 378487÷9.
8. 371561÷8.
9. 6897583÷5.
10. 4996504÷8.
11. 1918575÷6.
12. 5982039÷9.

How many times

13. Is 13 contained in 273? In 4550? In 36721?
14. Is 17 contained in 2465? In 50571? In 1299123?
15. Is 18 contained in 10404? In 7831? In 11052?
16. Divide 32760 by 12; by 13; by 14; by 15.
17. Divide 35280 by 25; by 32; by 27; by 24.
18. Divide 122764 by 36; by 24; by 28; by 26.
19. Divide 1970640 by 23; by 34; by 45; by 56.
20. Divide 7462450 by 42; by 36; by 52; by 48.
21. Divide 4523578 by 35; by 59; by 68; by 79.

REVIEW.

49. *1.* George has 54 cents, James has 30 cents, and Henry has 20 cents. How much more has George than each of the others? If George gives 15 cents to James and 23 cents to Henry, how much has each then?

2. 20 boys, in 20 days, earn 20 dollars each. How many dollars do they all earn?

3. A girl earns 36 dollars; she spends one-half for books and one-fourth for a dress. How much has she left?

4. There are 208 pages in a book; 28 lines on a page, and 11 words to a line. How many words in the book?

5. A man bought 3 barrels of flour. One cost him 8 dollars, another cost him 9 dollars, and another cost him 11 dollars. How much more would he have expended if he had paid 12 dollars for each barrel?

6. If a merchant gains 16 cents on each yard of cloth sold, what is his profit on 27 yards?

7. A ship sails 15 miles an hour for four days. How far does she sail?

8. A city has 3000 street lamps. If each lamp-lighter lights 75 lamps, how many lamp-lighters are required for the entire city?

9. How many pounds of tea worth 75 cents a pound, ought a man to receive in exchange for 27 bushels of oats, worth 50 cents a bushel?

10. A farmer sold 40 bushels of wheat at 2 dollars a bushel, and 16 cords of wood at 3 dollars a cord. He received 15 yards of cloth at 4 dollars a yard, and the remainder in money; how much money did he receive?

11. How many barrels of flour at 9 dollars a barrel can be bought for 175 dollars?

12. There are 320 rods in a mile. How many rods in half a mile? In one-fourth of a mile?

13. Two men start from the same place and travel in the same direction—one travels XXV miles a day; the other XVIII miles a day. How far apart are they in a week?

14. Bought 13 cows, at 40 dollars each; 16 sheep at 2 dollars each; a horse for 250 dollars, and a wagon for 97 dollars. How many months would it take to pay for them, in monthly payments of 18 dollars each?

15. A farmer raised 1500 bushels of oats, and sold them for 32 cents a bushel. How many acres of land could he buy, at 20 dollars an acre, with the proceeds?

16. A man received 574 dollars of interest in 7 years. What was that a year?

17. A man has an income from salary, of 75 dollars a month; from interest, of 28 dollars a month; from rents, of 62 dollars a month. It costs him 300 dollars a year for board and 18 dollars a year for taxes. How much will he have at the end of 2 years?

18. How many barrels of flour at 7 dollars a barrel, will pay for 30 tons of coal, at 4 dollars a ton, and 44 cords of wood, at 3 dollars a cord?

19. If a field containing 42 acres produce 1659 bushels of wheat, what will be the number of bushels per acre?

20. A farmer sold 15 tons of hay at 9 dollars a ton, and 25 cords of wood at 4 dollars a cord, and wished to divide the amount equally among 5 creditors; how much would each receive?

21. If 42864 pounds of cotton be packed in 94 equal bales, what is the weight of each bale?

22. If you deposit 216 cents each week in a savings bank, and take out 89 cents a week, how many cents will you have in bank at the end of 36 weeks?

23. A man sold 6 horses at 125 dollars each, 25 head of cattle at 30 dollars each, and with the proceeds bought land at 25 dollars an acre; how many acres did he buy?

24. A room is 40 feet wide, its length is twice its width, and its height, one-fifth its length. How high is it?

25. Bought 150 barrels of flour for 1050 dollars, and sold 107 barrels of it at 9 dollars a barrel, and the remainder at 7 dollars a barrel; did I gain or lose, and how much?

DRILL EXERCISES.

50. *1.* $\frac{36}{1}$, $\frac{36}{2}$, $\frac{36}{3}$, $\frac{36}{4}$, $\frac{36}{6}$, $\frac{36}{9}$, $\frac{36}{18}$, $\frac{36}{36}$.

Let the pupils recite in the following manner,—36 is 36 times 1, 18 times 2, 12 times 3, 9 times 4, etc. In like manner, treat each of the products of the multiplication table as given on Page 31.

2. Double, numbers as in exercises 2, 3 and 4, Page 32, and reverse the process by *halving* the final products; thus, Half of 8192, Half of 4096, etc.; the pupils naming the successive quotients.

3. 8, $\times 9$, $\div 12$, $+3$, $\times 5$, $+10$, $\div 11$, $\times 8$, $+20$, $\div 12$, $+4$, $\times 7$, $-3 =$?

Let the pupil read the successive results, as in the last exercise on Page 32.

4. $42 \div 7$, $\times 5$, $+6$, -8, $\div 4$, $\times 9$, -3, $\div 10$, $+12$, $\div 9$, $\times 11 =$?

DRILL TABLE No. 1.

	A.	B.	C.	D.	E.	F.	G.	H.
8.	2	3	7	9	8	4	1	6
7.	1	7	2	5	3	6	8	4
6.	4	2	6	3	7	5	9	1
5.	6	8	3	7	4	1	5	9
4.	3	5	8	8	6	3	7	2
3.	5	4	1	2	9	8	6	3
2.	8	6	5	4	1	2	3	7
1.	4	2	3	8	5	9	6	1

Add each column from bottom to top. Add each column from top to bottom. Add each line from left to right. Add each line from right to left.

Start with 1 and add to it each column from top to bottom. Start with 3, 4, etc., to 9. Same from bottom to top. From right to left. From left to right. This affords over 1000 different exercises in addition.

To 1 add each number in column A. To 2, 3, etc., to 10. Same with each column. By columns give the sum of each number and the one below it. By lines give the sum of each number and the one to right of it.

By column give the difference of each number and the one below it. By lines give difference of each number and one to right of it. In column A, give difference of each number and 1. 2, 3, etc., to 20. Same with each column.

By column give product of each number and the one to right of it. In column A, give product of each number and 2. Also 3, 4, etc., to 9. Same with each column.

Divide 10 by each number in column A, naming only quotient and remainder. Also 11, 12, etc., to 25. Same with each column.

Begin at the left and divide the product of any two numbers in columns A and B by the number at the right in column C; then of the columns B and C by the numbers in column D, etc. In the same manner begin with the two columns on the right.

Also beginning at the top, divide the product of the numbers in any two lines by the number in next line below; then in same manner begin with the two lines at the bottom, and divide by the numbers in the next line above.

This table and similar tables, placed upon the blackboard, will afford abundant exercise for months. *In no case should more than five minutes daily be given to this kind of work.*

FRACTIONS.

51. A ***Unit*** is one, or a single thing. **(Art. 1.)**

52. Numbers that express or represent *equal parts* of a unit or whole thing are called ***Fractions.***

A fraction is usually expressed by two numbers, called the *Numerator* and *Denominator,* one written over the other with a line between them. When so written, they are sometimes called *Common Fractions;* thus,

One-half	is written	$\frac{1}{2}$	Five-sixths	is written	$\frac{5}{6}$
One-third	"	$\frac{1}{3}$	Nine-tenths	"	$\frac{9}{10}$
Three-fourths	"	$\frac{3}{4}$	Seven-twentieths	"	$\frac{7}{20}$

53. The number below the line is the ***Denominator.***

Thus, in the fraction $\frac{3}{4}$, 4 is the *denominator* or *namer,* and shows that the unit is divided into *four* equal parts, named *fourths.*

54. The number above the line is the ***Numerator.***

Thus, in $\frac{3}{4}$, 3 is the *numerator* or *numberer,* and shows that 3 of the 4 equal parts *are taken,* or expressed, by the fraction.

55. The *numerator* and *denominator* of a fraction are called the ***Terms of the Fraction.***

Thus, 3 and 4 are *terms* of the fraction $\frac{3}{4}$.

Read the following fractions, and name the *terms:*

$\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{4}{7}$, $\frac{5}{8}$, $\frac{4}{5}$, $\frac{5}{9}$, $\frac{1}{8}$, $\frac{2}{7}$, $\frac{3}{10}$, $\frac{7}{8}$.

Write the following in figures:

Three-sevenths.
Two-fifths.
Three-eighths.
Seven-ninths.
Seven-tenths.
Five-sevenths.
Five-elevenths.
Nine-fourteenths.
Eleven-twelfths.

56. *1.* How do you find *one-half* of any number?
Ans. Divide the number by 2.

2. How do you find 1 third of any number? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? etc.

3. What is $\frac{1}{2}$ of 6? Of 8? Of 12? Of 18? Of 20?

4. What is $\frac{1}{3}$ of 9? Of 15? Of 21? Of 24? Of 27?

5. What is $\frac{1}{4}$ of 12? Of 16? Of 28? Of 32? Of 40?

6. What is $\frac{1}{5}$ of 20? Of 35? Of 45? Of 50? Of 60?

7. What is $\frac{1}{6}$ of 18? $\frac{1}{7}$ of 21? $\frac{1}{5}$ of 30? $\frac{1}{4}$ of 36?

8. What is $\frac{1}{7}$ of 28? $\frac{1}{6}$ of 54? $\frac{1}{8}$ of 48? $\frac{1}{7}$ of 63?

9. What is $\frac{1}{8}$ of 40? $\frac{1}{9}$ of 45? $\frac{1}{7}$ of 49? $\frac{1}{9}$ of 72? $\frac{1}{8}$ of 56?

10. What do you understand by $\frac{2}{5}$ of any number or thing? *Ans.* *Two* of the *five equal parts* into which the number or thing is divided.

11. What do you understand by $\frac{4}{5}$ of any number or thing? By $\frac{3}{4}$? By $\frac{1}{6}$? $\frac{5}{6}$? By $\frac{1}{7}$? $\frac{3}{7}$? $\frac{5}{7}$? By $\frac{1}{8}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$?

12. What is meant by $\frac{1}{9}$ of any number or thing? By $\frac{2}{9}$? $\frac{4}{9}$? $\frac{5}{9}$? $\frac{7}{9}$? By $\frac{1}{10}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$?

13. What is $\frac{2}{3}$ of 12?

ANALYSIS —*One*-third of 12 is 4; 2 thirds of 12 are 2 times 4, or 8. Hence $\frac{2}{3}$ of 12 are 8.

14. What is $\frac{3}{4}$ of 16? $\frac{2}{3}$ of 15? $\frac{2}{5}$ of 20? $\frac{4}{5}$ of 25?

15. What is $\frac{2}{6}$ of 18? $\frac{5}{6}$ of 30? $\frac{3}{7}$ of 21? $\frac{4}{7}$ of 28? $\frac{3}{8}$ of 32? $\frac{3}{4}$ of 36?

16. What is $\frac{2}{9}$ of 27? $\frac{4}{9}$ of 18? $\frac{5}{9}$ of 36? $\frac{1}{10}$ of 40? $\frac{7}{10}$ of 30? $\frac{3}{11}$ of 44?

17. What is $\frac{3}{7}$ of 28? $\frac{5}{6}$ of 60? $\frac{5}{8}$ of 64? $\frac{2}{9}$ of 63? $\frac{7}{10}$ of 90? $\frac{1}{12}$ of 72?

57. *1.* How do you find how many *halves* there are in any whole number? *Ans.* Multiply the whole number by 2.

2. How do you find how many *thirds* there are in any number? How many *fourths?* *Fifths?* *Sixths?* *Sevenths?* *Eighths?* *Ninths?* *Tenths?* etc.

3. How many halves are there in 3 peaches?

ANALYSIS.—Since in 1 peach there are 2 *halves*, in 3 peaches there are 3 times 2 *halves*, or 6 *halves*. Hence in 3 peaches are 6 *halves*.

4. How many *halves* are there in 4? In 8? In 9?

5. How many *thirds* are there in 5? In 7? In 8?

6. How many *fourths* are there in 7? *fifths* in 8? *sixths* in 6? *sevenths* in 9? *eighths* in 7? *ninths* in 5? *tenths* in 8?

7. How many *halves* in 1 and 1 half?

ANALYSIS.—In 1 are 2 *halves*, and 1 *half* added make 3 *halves*. Hence, in 1 and 1 half there are 3 *halves*.

8. How many *halves* in 3 and 1 half? In 5 and 1 half?

9. How many *thirds* in 2 and 2 thirds?

ANALYSIS.—Since in 1 there are 3 thirds, in 2 there are 2 times 3 thirds, or 6 thirds, and 2 thirds added make 8 thirds. Hence, in 2 and 2 thirds there are 8 *thirds*.

10. How many *thirds* in 4 and 1 third? *fourths* in 3 and 1 fourth?

11. How many *fifths* in 2 and 3 fifths? *sixths* in 5 and 4 sixths? *sevenths* in 3 and 2 sevenths?

12. How many *fourths* in 6 and 3 fourths? In 7 and 1 fourth? In 8 and 3 fourths?

13. How many *sixths* in 4 and 1 sixth? In 5 and 5 sixths? *ninths* in 6 and 4 ninths?

58. When the numerator and denominator of a fraction are equal, the *value* of the fraction is equal to 1; as $\frac{3}{3}=1$; $\frac{4}{4}=1$; $\frac{6}{6}=1$, etc.

59. When the numerator is greater than the denominator, the *value* is greater than 1; as $\frac{4}{3}=1\frac{1}{3}$; $\frac{8}{4}=2$, etc.

60. A whole number and a fraction written together are called a ***Mixed Number;*** as $5\frac{3}{4}$, read 5 and 3 fourths.

1. Read the following *mixed numbers:*

$3\frac{5}{8}$; $9\frac{2}{3}$; $7\frac{5}{6}$; $10\frac{4}{5}$; $8\frac{7}{9}$; $12\frac{7}{10}$; $14\frac{7}{12}$; $5\frac{1}{9}$.

2. How many *fifths* are $4\frac{3}{5}$?

ANALYSIS.—Since in 1 there are 5 fifths, in 4 there are 4 times 5 fifths, or 20 fifths, and 3 fifths added make 23 fifths. Hence $4\frac{3}{5}$ are $\frac{23}{5}$.

3. How many *fourths* are $9\frac{1}{4}$? Are $7\frac{3}{4}$? Are $8\frac{2}{4}$?

4. How many *thirds* are $12\frac{2}{3}$? *Fifths* are $6\frac{4}{5}$? *Sixths* are $7\frac{5}{6}$? *Fourths* are $10\frac{3}{4}$?

5. Change $4\frac{5}{8}$ to *eighths;* $9\frac{3}{7}$ to *sevenths;* $8\frac{7}{10}$ to *tenths;* $5\frac{7}{9}$ to *ninths;* $4\frac{5}{12}$ to *twelfths.*

6. How many *ones* are $\frac{12}{4}$?

ANALYSIS.—Since 4 fourths equal 1, 12 fourths are as many 1's as 4 fourths are contained times in 12 fourths, or 3 times. Hence $\frac{12}{4}$ are 3 *ones* or 3. Or $\frac{12}{4}=12\div4=3$.

7. Find the value of $\frac{15}{3}$; of $\frac{18}{6}$; of $\frac{21}{7}$; of $\frac{28}{4}$; of $\frac{36}{9}$.

8. Change to a whole or a mixed number $\frac{30}{5}$; $\frac{27}{6}$; $\frac{36}{5}$; $\frac{42}{7}$; $\frac{50}{10}$; $\frac{45}{8}$; $\frac{37}{4}$; $\frac{60}{12}$.

9. What is the value of $\frac{44}{8}$? $\frac{56}{7}$? $\frac{72}{12}$? $\frac{60}{8}$? $\frac{57}{10}$? $\frac{70}{9}$?

10. Find the value of $\frac{75}{9}$; $\frac{66}{11}$; $\frac{84}{9}$; $\frac{96}{12}$; $\frac{100}{10}$.

11. Change to a whole or mixed number $\frac{90}{8}$; $\frac{75}{12}$; $\frac{63}{5}$; $\frac{84}{12}$; $\frac{100}{9}$; $\frac{86}{11}$; $\frac{120}{10}$.

12. Find the value of $\frac{67}{9}$; $\frac{88}{12}$; $\frac{90}{11}$; $\frac{117}{10}$; $\frac{150}{15}$.

61. *1.* How many are 2 *fifths* and 3 *fifths?*

Ans. 5 *fifths,* or $\frac{5}{5}=1$.

2. Can you add 3 *apples* and 5 *figs?* Why not?

3. Can you add 3 *fourths* and 5 *sixths?* Why not?

4. Only fractions having the *same denominator* can be added.

5. How many are $\frac{7}{8}+\frac{3}{8}$? *Ans.* $\frac{10}{8}=1\frac{2}{8}$.

6. What is the sum of $\frac{5}{6}$ and $\frac{3}{6}$? Of $\frac{7}{10}$ and $\frac{6}{10}$? Of $\frac{5}{9}$ and $\frac{7}{9}$? Of $\frac{9}{10}$ and $\frac{4}{10}$?

7. Add $\frac{5}{12}$ and $\frac{9}{12}$. $\frac{7}{11}$ and $\frac{8}{11}$. $\frac{4}{8}$ and $\frac{7}{8}$. $\frac{2}{8}$ and $\frac{5}{8}$.

8. How many are 7 *ninths* less 2 *ninths?*

Ans. 5 *ninths,* or $\frac{5}{9}$.

9. Can you take 6 *books* from 9 *slates?* Why not?

10. Can you subtract 6 *sevenths* from 9 *twelfths?* Why not?

11. One fraction can be subtracted from another, only when both have the *same denominator.*

12. How many are $\frac{7}{9}-\frac{3}{9}$? $\frac{9}{10}-\frac{7}{10}$? $\frac{7}{12}-\frac{1}{12}$? $\frac{11}{14}-\frac{9}{14}$?

13. Subtract $\frac{2}{3}$ from $\frac{5}{3}$; $\frac{2}{11}$ from $\frac{9}{11}$; $\frac{4}{15}$ from $\frac{13}{15}$.

14. How many are $\frac{3}{7}+\frac{4}{7}+\frac{5}{7}+\frac{6}{7}$? *Ans.* $\frac{18}{7}=2\frac{4}{7}$.

15. How many 1's are $\frac{5}{9}+\frac{1}{9}+\frac{7}{9}+\frac{8}{9}+\frac{10}{9}$?

16. From $\frac{5}{8}+\frac{3}{8}+\frac{7}{8}$ take $\frac{2}{8}$. *Ans.* $\frac{13}{8}=1\frac{5}{8}$.

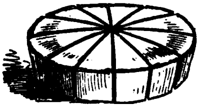
17. From $\frac{7}{12}+\frac{5}{12}+\frac{11}{12}$ subtract $\frac{8}{12}$.

18. John spent $\frac{3}{8}$ of his money and lost $\frac{2}{8}$ of it. What part of his money had he left?

19. James paid $\frac{5}{8}$ of a dollar for a book, and $\frac{3}{8}$ of a dollar for a slate. How much did he pay for both? How much more for the book than for the slate?

20. If Lillie has 3 fourths of an orange and Clarence the remainder, how much has Clarence?

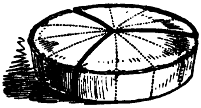
62. *1.* If a cake is cut into twelve equal pieces, what part of the whole cake is each piece? *Ans. One-twelfth.*



2. How many *twelfths* make a whole thing?

3. How do we find 1 *twelfth* of any number?

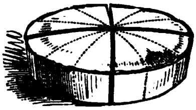
4. If a cake is divided into 3 equal parts, each part is called 1 *third.*



5. If each *third* of the cake is divided into 4 equal parts, into how many parts is the *whole cake* divided?

6. How many *twelfths* in 1 third? In 2 thirds? 3 thirds?

7. If a cake is cut into 4 equal parts, each part is called 1 *fourth.*



8. If each *fourth* of the cake is divided into 3 equal parts, into how many parts is the *whole cake* divided?

9. How many *twelfths* in 1 fourth? In 2 fourths? In 3 fourths? In 4 fourths?

10. If Edie has $\frac{1}{3}$ of a cake and Asa $\frac{1}{4}$ of it, what part have both? $\frac{1}{3}+\frac{1}{4}=?$

ANALYSIS.—Since $\frac{1}{3}$ is equal to $\frac{4}{12}$, and $\frac{1}{4}$ is equal to $\frac{3}{12}$, they have the *sum* of $\frac{4}{12}$ and $\frac{3}{12}$, or $\frac{7}{12}$.

11. How much more has Edie than Asa? $\frac{1}{3}-\frac{1}{4}=?$

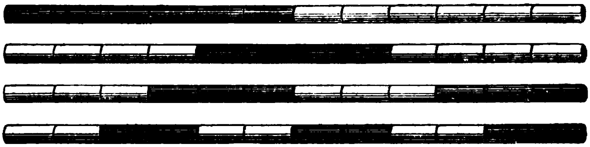
12. How many twelfths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{2}{3}$? In $\frac{1}{4}$? In $\frac{3}{4}$?

13. Which is greater, $\frac{1}{3}$ or $\frac{1}{4}$? $\frac{1}{2}$ or $\frac{1}{3}$? $\frac{2}{3}$ or $\frac{3}{4}$?

14. If Mary has $\frac{1}{2}$ and Jennie $\frac{1}{4}$ of a melon, what part of the whole melon have both?

15. A farmer sold $\frac{1}{3}$ of his grain to one man, $\frac{1}{4}$ to another; what part of all his grain did he sell?

63. *1.* Draw lines upon the slate or board of equal length, and divide them into twelve equal parts.



2. By a heavier mark these lines may be divided into *halves*, *thirds*, *fourths*, and *sixths*, respectively.

$1 = \frac{12}{12}$ $\frac{1}{3} = \frac{4}{12}$ $\frac{1}{6} = \frac{2}{12}$

$\frac{1}{2} = \frac{6}{12}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{1}{12} = \frac{1}{12}$

3. Hence, it is plain that halves, thirds, fourths, sixths, and twelfths, may be added or subtracted when changed to parts of *equal size* or *magnitude*.

What is the sum of

4. $\frac{1}{2}$ and $\frac{2}{3}$?
5. $\frac{1}{3}$ and $\frac{1}{4}$?
6. $\frac{3}{4}$ and $\frac{1}{6}$?
7. $\frac{2}{3}$ and $\frac{3}{4}$?
8. $\frac{5}{6}$ and $\frac{3}{12}$?
9. $\frac{2}{6}$ and $\frac{3}{4}$?
10. $\frac{1}{6}$ and $\frac{1}{2}$?
11. $\frac{3}{12}$ and $\frac{3}{4}$?
12. $\frac{2}{3}$ and $\frac{1}{6}$?
13. $\frac{1}{2}$ and $\frac{5}{8}$?
14. $\frac{3}{4}$ and $\frac{3}{8}$?
15. $\frac{1}{3}$ and $\frac{5}{9}$?

What is the value of

16. $\frac{1}{2} - \frac{1}{3}$?
17. $\frac{1}{2} - \frac{1}{4}$?
18. $\frac{1}{3} - \frac{1}{6}$?
19. $\frac{3}{4} - \frac{2}{3}$?
20. $\frac{2}{3} - \frac{1}{6}$?
21. $\frac{5}{6} - \frac{2}{3}$?
22. $\frac{1}{4} - \frac{1}{6}$?
23. $\frac{7}{12} - \frac{3}{6}$?
24. $\frac{3}{4} - \frac{9}{12}$?
25. $\frac{3}{4} - \frac{1}{8}$?
26. $\frac{7}{8} - \frac{1}{2}$?
27. $\frac{2}{3} - \frac{4}{9}$?

When fractions have different denominators, they must be changed to fractions of like value, having the same denominators, before they can be added or subtracted.

64. *1.* 4 is $\frac{1}{2}$ of what number?

ANALYSIS.—Since 4 is 1 half of a number, 2 halves or the number are 2 times 4, or 8. Hence 4 is $\frac{1}{2}$ of 2 times 4, or 8.

2. 5 is $\frac{1}{3}$ of what number? $\frac{1}{4}$ of what number?

3. 7 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number?

4. 6 is $\frac{1}{4}$ of what number? $\frac{1}{5}$ of what number?

5. 4 is $\frac{1}{8}$ of what number? $\frac{1}{6}$ of what number?

6. 8 is $\frac{1}{5}$ of what number? $\frac{1}{4}$ of what number?

7. 3 is $\frac{1}{9}$ of what number? $\frac{1}{10}$ of what number?

8. If $\frac{1}{3}$ of a ton of coal cost 2 dollars, what will 1 ton cost?

9. George gave a beggar 6 cents, which was $\frac{1}{5}$ of all the money he had; how much money had he?

10. If $\frac{1}{4}$ of a pound of coffee cost 8 cents, what will $\frac{3}{4}$ of a pound cost?

ANALYSIS.—Since 1 fourth of a pound costs 8 cents, 3 fourths of a pound will cost 3 times 8 cents, or 24 cents.

11. If $\frac{1}{6}$ of a bushel of pears cost 9 cents, what will $\frac{5}{6}$ of a bushel cost?

12. In $\frac{1}{10}$ of a dollar are 10 cents; how many cents in $\frac{7}{10}$ of a dollar?

13. If a man walk 6 miles in $\frac{1}{4}$ of a day, how many miles can he walk in 1 day?

14. A pencil cost 8 cents, which was $\frac{1}{6}$ the cost of a book; what was the cost of the book?

15. If $\frac{1}{8}$ of a melon cost 7 cents, what will $\frac{5}{8}$ of it cost?

16. If $\frac{1}{6}$ of a barrel of flour is worth 2 dollars, what is $\frac{5}{6}$ of a barrel worth? What is a whole barrel worth?

17. A garden is 4 rods wide, which is $\frac{1}{5}$ its length; what is its length?

65. *1.* If a barrel of sugar is worth 24 dollars, what is $\frac{1}{3}$ of it worth? 2 thirds? 1 fourth? 3 fourths?

2. If a boy earns 10 dollars in a week, and spends 2 fifths of it, what part is left? How many dollars?

3. When coal is 7 dollars a ton, what part of a ton will 1 dollar buy? 2 dollars? 3 dollars? 5 dollars?

4. At $\frac{2}{3}$ of a dollar a yard, what will 4 yards of cloth cost?

ANALYSIS.—Since 1 yard costs $\frac{2}{3}$ of a dollar, 4 yards will cost 4 times $\frac{2}{3}$ or $\frac{8}{3}$ of a dollar, equal to $2\frac{2}{3}$ dollars. Hence 4 yards of cloth cost $2\frac{2}{3}$ dollars.

5. At $\frac{2}{5}$ of a dollar a pound, what will 6 pounds of butter cost? 8 pounds? 9 pounds?

6. How many are 3 times $\frac{4}{5}$? 2 times $\frac{7}{8}$? 4 times $\frac{1}{9}$?

7. How many are 5 times $\frac{2}{7}$? 6 times $\frac{3}{4}$? 7 times $\frac{1}{8}$?

8. What is the difference between 3 fourths of 24 and 4 fifths of 30? 2 thirds of 27 and 1 sixth of 60?

9. If a ton of hay cost 21 dollars, what will 3 sevenths of a ton cost? 2 thirds of a ton?

10. A boy having 20 marbles, gave $\frac{1}{4}$ of them to one companion, and $\frac{1}{5}$ to another; how many had he left?

11. Jane paid 25 cents for a book, and $\frac{4}{5}$ as much for a slate; what did she pay for the slate?

12. Fanny is 14 years old, and her sister is $\frac{4}{7}$ as old; how old is her sister?

13. What will be the cost of 6 boxes of figs, at $\frac{3}{8}$ of a dollar a box?

14. If 1 pineapple cost $\frac{2}{5}$ of a dollar, what will 4 pine apples cost? What will 6 cost? 8? 9? 10?

15. If a basket hold $\frac{5}{6}$ of a bushel, how much will 4 baskets hold? 6 baskets? 7 baskets?

66. *1.* How many oranges in $\frac{6}{3}$ of an orange? In 7 thirds? In 9 thirds? In 10 thirds?

2. How many yards in $\frac{7}{5}$ of a yard? In $\frac{8}{5}$ of a yard? In $\frac{10}{5}$? In $\frac{12}{5}$? In $\frac{15}{5}$?

3. If 3 books cost 12 fifths of a dollar, what will 1 book cost?

ANALYSIS.—Since 3 books cost 12 fifths of a dollar, 1 book will cost 1 third of 12 fifths, or 4 fifths of a dollar. Hence 1 book costs $\frac{4}{5}$ of a dollar.

4. If 3 pounds of coffee cost $\frac{9}{10}$ of a dollar, what will 1 pound cost? What will 5 pounds cost?

5. If 4 yards of ribbon cost $\frac{8}{9}$ of a dollar, what will 3 yards cost? 5 yards? 7 yards? 10 yards?

6. If a man walks $\frac{2}{3}$ of 6 miles in 1 hour, how far can he walk in 3 hours? How far in $\frac{1}{3}$ of an hour?

7. If 5 bushels of oats are worth $\frac{15}{4}$ of a dollar, what is 1 bushel worth? 3 bushels? 6 bushels? 10 bushels?

8. If 3 fourths of a bushel of cranberries are worth 3 dollars, what is $\frac{1}{2}$ of 6 bushels worth?

9. At 4 dollars a yard, what will $2\frac{1}{2}$ yards of cloth cost?

ANALYSIS.—Since 1 yard costs 4 dollars, $2\frac{1}{2}$ yards will cost $2\frac{1}{2}$ times 4 dollars; 2 times 4 dollars are 8 dollars, and $\frac{1}{2}$ of 4 dollars is 2 dollars, which added to 8 dollars make 10 dollars. Hence $2\frac{1}{2}$ yards will cost 10 dollars.

10. At 10 dollars a barrel, what will $\frac{1}{2}$ barrel of flour cost? What will $\frac{3}{5}$ of a barrel cost? $2\frac{1}{5}$ barrels?

11. At $\frac{3}{4}$ of 16 cents a pound, what will $\frac{1}{2}$ of a pound of sugar cost? $\frac{1}{3}$ of a pound? $1\frac{1}{2}$ pounds? $2\frac{1}{4}$ pounds?

12. At 15 dollars a ton, what will $\frac{2}{3}$ of a ton of hay cost? $\frac{3}{4}$ of a ton? $1\frac{1}{3}$ tons? $1\frac{2}{3}$ tons?

MEASURES.

67. In measuring a quantity, some *definite part* of it is taken as a ***Unit of Measure;*** as, a yard, a gallon, etc., by which all quantities of *that kind* are measured.

The *length* of a piece of cloth is ascertained by applying a *yard* measure; the *capacity* of a cask by the use of a *gallon* measure; the *weight* of a body by the *pound* weight, etc.

68. Measures may be classified into six kinds: Money or Value, Extension, Capacity, Weight, Time, Angles or Arcs.

MONEY.

69. ***Money*** is the measure of the *value* of things.

70. The legal money of this country is called ***United States Money.***

The ***Gold Dollar*** is the *unit* of United States Money.

TABLE.

10 Mills (*m.*) = 1 Cent . . *ct.*
10 Cents = 1 Dime . . *d.*
10 Dimes = 1 Dollar . *$.*

$$\$1 = \begin{cases} 1000\ m. \\ 100\ ct. \\ 10\ d. \end{cases}$$

100 cents = 1 dollar; 75 cents = ¾ dollar; 50 cents = ½ dollar;
25 cents = ¼ dollar; 20 cents = ⅕ dollar; 12½ cents = ⅛ dollar.

1. The money of the Dominion of Canada is the same as that of the United States.
2. The value of a *Sovereign* in United States Money is $4.8665.
3. The value of a *Franc* in United States Money is $.193.
4. The value of a Reichsmark ("*Mark*") in U. S. Money is $.238.

71. *1.* How many mills in 1 cent? 3 cents? 5 cents?

2. How many cents are 30 mills? 50 mills? 60 mills?

3. How many dimes are 20 cents? 40 cents?

4. How many cents are 3 dimes? 4 dimes? 6 dimes?

5. How many dimes in 1 dollar? How many cents?

6. In half a dollar, how many dimes? Cents?

7. How many cents are 1 dollar? 2 dollars?

8. How many cents in 1½ dollars? 2¼ dollars?

9. How many dollars in 200 cents? In 150? In 450?

The *sign* $ signifies ***dollar*** or ***dollars,*** and is placed before the number. Thus $3 means *three dollars;* $14 means 14 *dollars,* etc.

When *dollars* and *cents* are written together, they are separated by a point (.); thus, $3.75 means 3 *dollars* and 75 *cents.*

Since 100 cents make $1.00, *cents* always occupy *two* places, and never more than two.

When *cents* alone are expressed, the dot is placed after the sign $ and before the number; thus, $.50 is 50 cents; $.18 is 18 cents, etc. Or we may write the word *cents* or *cts.* after the number; thus, 50 *cents.*

Mills are written after cents; as, $4.375 is 4 dollars 37 cents 5 mills. 5 mills are ½ cent; $4.375 = $4.37½.

Read the following:

$3.46
$4.50
$7.62
$.375
$6.08
$.427
$15.03
$10.10
$25.625
$1.12½
$.796
$.075

When the cents are less than 10, a cipher must be placed before them and after the dot; thus, *eight* cents is written 8 cents, or $.08; *six* cents, 6 cents, or $.06, etc.

Exercises for the Slate and Board.

72. Write in figures

1. Forty-four dollars and twenty-six cents.

2. Eighteen dollars and seventy-five cents.

3. Thirty-five cents.

4. Eighty-seven cents.

5. Sixty cents.

6. Fifteen cents.

7. Nine cents.

8. Twenty cents.

9. Seventy-eight dollars sixty-two cents and five mills.

10. One hundred fourteen dollars and ninety cents.

11. In adding and subtracting, dollars should be placed under dollars, and cents under cents, so that the dots may be in the same column.

Add the following:

(*12.*)	(*13.*)	(*14.*)	(*15.*)
$3.50	$19.37	$6.29	$.875
12.48	.84	23.82	1.065
.75	5.09	1.10	12.63

Arrange in columns and add:

16. $11.36, $26.07, $9.16, $32.76, and $2.34.

17. $42.06, $10.30, $4.82, $.77, and $.93.

Subtract the following:

	(*18.*)	(*19.*)	(*20.*)	(*21.*)
From	$17.48	$73.26	$50.67	$120.80
Take	6.27	25.18	10.08	35.26

Find the value of

22. $57.10—$12.40.

23. $104.47—$73.92.

24. $100.375—$40.095.

25. $416.08—$208.67.

26. $57.46—$18.00+$24.25=?

27. $50.20+$76.31—$4.01=?

28. $98.76—$30.47+$43.04=?

29. $145.09+$.86—$75=?

Find the sum

30. Of $32.50, $126.085, $9.408, $15.74, and $140.

31. Of $307.09, $50, $6.848, $100.10, and $450.

32. Of $76, $400, $5.125, $17.04, $.975, and $1.625.

33. A lady paid $45.40 for a dress, $15.37½ for a bonnet, $6 for a pair of gaiters, and $1.625 for a pair of gloves. What did she pay for all?

34. A farmer sold a cow for $36.50, a ton of hay for $14.25, and a tub of butter for $20.80. What did he receive for all?

35. Bought a hat for $4.75, a pair of shoes for $5.125, a pair of gloves for $.87½, and an umbrella for $2.75. What was the cost of the whole?

Find the difference between

36. $46.75 and $14.45.

37. $142.09 and $68.36.

38. $300.085 and $104.50.

39. $87.875 and $5.10.

40. $250 + $2.75 and $124.50.

41. $617.10 + $50.125 and $10.37½.

42. $908.46 and $325 + $5.25.

43. A man bought a horse for $150, and sold him for $137.50. What did he lose?

44. A grocer paid $28.75 for a barrel of sugar and sold it for $34. What did he gain?

45. Bought a sack of flour for $1.75, a pound of tea for $.90, and some sugar for $2.25. How much must I receive in change for a 5-dollar bill.

46. Paid $450 for a pair of horses, and sold one of them for $275.50. What did the other cost me?

47. Two men bought a piece of property for $358.50. One paid $146.30; how much did the other pay?

Find the value

49. Of $93.67 − $12.80.

50. Of $118.19 − $9.87½.

51. Of $500 − $300.277.

52. Of $76.84½ − $40.12½.

53. Of $125 − $75 + $14.50.

54. Of $9.10 + $46.08 − $25.

55. Of $48 − $12.50 − $13.92.

56. Of $310 − $7.10 − $200.

57. A farmer sold a ton of hay for $12.50, and a cord of wood for $3.25. He received in payment a barrel of flour worth $7.60, and the remainder in money. How much money did he receive?

58. A grocer sold some tea for 80 cents, some butter for 2 dollars 30 cents, some eggs for 53 cents, and some sugar for one dollar and ten cents. How much change should he return for a five-dollar bill?

59. Mary went shopping and had 2 five-dollar bills; she bought a dress for 7 dollars 25 cents, trimmings for 2 dollars 37½ cents, some thread for 12½ cents, and some tape and needles for twenty cents. How much money had she left?

60. Henry gave one dollar and a half for a pair of skates, seventy-five cents for a cap, thirty-seven cents for a ball, half a dollar for a knife, two dollars and a quarter for a sled, and had one dollar left. How much money had he at first?

61. A man owed $427.50. He paid at one time $125.75, at another $100, and at another $50.25. What remained unpaid?

62. James had $5.48, Henry had $1.17 more than James, and George had $.75 less than James and Henry together. How much money had George; and how much had they all?

73. *1.* At $5.75 a ton, what will 5 tons of coal cost?

$5.75
5
$28.75

ANALYSIS.—Since 1 ton costs $5.75, 5 tons will cost 5 times $5.75, or $28.75.

When the multiplicand contains *cents*, put the point (.) in the product *two* places from the *right*, and prefix the sign ($) to the whole.

	(64.)	*(65.)*	*(66.)*	*(67.)*
Multiply	$24.32	$42.09	76 cents.	$1.87
By	4	5	8	6
Product	$97.28	$210.45	$6.08	$11.22

In like manner,

2. Multiply $326 by 5; by 7; by 8; by 9; by 12.

3. Multiply $64.25 by 6; by 8; by 7; by 9.

4. Multiply $85.36 by 12; by 14; by 16; by 24.

5. Multiply $248.08 by 25; by 26; by 34; by 42.

Find the cost

6. Of 7 barrels of flour, at $8.60 a barrel.

7. Of 22 yards of cloth, at $4.35 a yard.

8. Of 40 bushels of wheat, at $1.75 a bushel.

9. Of 13 pounds of tea, at $1.10 a pound.

10. Of 27 pounds of butter, at 33 cents a pound.

11. Of 34 barrels of potatoes, at $3.75 a barrel.

12. At $4.15 a box, what are 18 boxes of oranges worth?

13. At $.875 a pound, what are 16 pounds of tea worth?

14. A farmer sold 6 cords of wood at $4.25 a cord, and 18 barrels of apples at $3.15 a barrel. What did he receive for both?

15. A merchant bought 25 yards of cloth at $3.50 a yard, and sold it for $4.15 a yard. What was his gain?

74. *1.* **If 6 pounds of tea cost $7.50, what costs 1 pound?**

Divisor. Dividend.
6) $ 7 . 5 0
Quotient, $ 1 . 2 5

ANALYSIS.—Since 6 pounds cost $7.50, 1 pound costs 1 *sixth* of $7.50, or $1.25.

When the dividend contains *cents*, put the point (.) in the quotient *two* places from the *right*, and prefix the sign ($) to the whole.

In like manner divide the following:

(*2.*)	(*3.*)	(*4.*)	(*5.*)
8) $124.96	7) $6.58	5) $19.70	9) $376.02
$15.62	$.94	$3.94	$41.78

6. Divide $705.60 by 4; by 5; by 6; by 7.

Find
7. 1 *fifth* of $461.50.
8. 1 *eighth* of $17.36.
9. 1 *seventh* of $243.04.
10. 1 *sixth* of $500.10.

What is
11. $\frac{1}{8}$ of 67344 inches?
12. $\frac{1}{9}$ of 437868 feet?
13. $\frac{1}{5}$ of 134860 pounds?
14. $\frac{1}{7}$ of $1046.85?

15. Paid $57.75 for 8 sheep. What did each cost?

16. If 6 tons of coal cost $40.80, what costs 1 ton?

17. Paid $9.88 for 13 pounds of tea. What did 1 pound cost? 7 pounds? 9 pounds? 12 pounds?

Find the value
18. Of $35.50 ÷ 4 × 12.
19. Of $4.35 ÷ 3 × 30.
20. Of $56.80 ÷ 8 × 19.
21. Of $90 × 18 ÷ 12.
22. Of $6.26 × 24 ÷ 8.
23. Of $12.75 × 14 ÷ 7.

24. If 5 boxes of lemons are worth $23.25, what are 14 boxes worth?

25. If 6 books cost $1.90, what will 27 books of the same kind cost?

75. ***Extension*** has one or more of the dimensions, *length*, *breadth*, and *thickness*. It may be a *line*, a *surface*, or a *solid*.

76. A ***Line*** has only one dimension—*length*.

77. ***Linear Measure,*** called also ***Long Measure,*** is used in measuring *lines*, or distances.

TABLE.

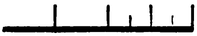
12 Inches (*in*)	= 1 Foot . . . *ft.*	1 *Mi.* =	63360 *in.*
3 Feet	= 1 Yard . . . *yd.*		5280 *ft.*
5½ Yards, or 16½ Ft.	= 1 Rod . . . *rd.*		1760 *yd.*
320 Rods	= 1 Mile . . . *mi.*		320 *rd.*

In measuring goods sold by the *yard*, the yard is divided into *halves*, *fourths*, and *eighths*.

OTHER DENOMINATIONS.

4 Inches	= 1 Hand,	Used to measure height of horses.
6 Feet	= 1 Fathom,	" " depth at sea.
1.15 Statute Miles	= 1 Geog. Mi.	" " distances "
3 Geographical Miles	= 1 League.	
69.16 Common Miles	= 1 Degree.	
360 Degrees	= The Circumference of the Earth.	

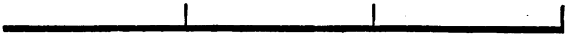
1. The *units* of linear measure are *lines*.

2. This *line*  is *one inch* long.

3. The inch is divided into *halves*, *fourths*, *eighths*, etc.

4. Make a line *twice* as long; *three times* as long.

5. How many inches long is this line?



6. How many inches in a line 1 foot long? In 1 ft. 6 in.? In 2 ft.? In 2 ft. 8 in.?

7. How many ft. in 1 yd.? How many inches?

8. How many feet in 3 yd. 2 ft.? In 5½ yd.? In 2 fathoms?

78. *Surface* or *Square Measure* is used in measuring *surfaces;* as of land, boards, plastering, etc.

TABLE.

144 Square Inches (*sq. in.*) = 1 Square Foot . . *sq. ft.*
9 Square Feet = 1 Square Yard . . *sq. yd.*
30¼ Square Yards = 1 Sq. Rod or Perch. *sq. rd.*; *P.*
160 Square Rods = 1 Acre . . . . . *A.*
640 Acres = 1 Square Mile . . *sq. mi.*
36 Square Miles = 1 Township . . . *Tp.*

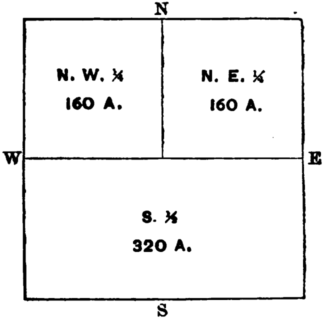
sq. mi. *A.* *sq. rd.* *sq. yd.* *sq. ft.* *sq. in.*
1 = 640 = 102400 = 3097600 = 27878400 = 4014489600

The square mile is frequently called a *Section*, and is subdivided into *half-sections, quarter-sections*, etc.

1. How many acres are there in a section? How many in a half-section? In a quarter-section?

2. How many acres in the N. E. quarter-section and the S. half-section?

A SECTION.



3. A man owns one-half of the N. W. quarter-section. How much land has he?

4. In 2⅔ square yards how many square feet?

5. How many square rods in 40 acres and 40 square rods? In 25¼ acres?

6. How many square inches in 75 square feet?

7. How many acres in a township?

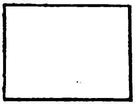
8. How many square rods in 7½ acres?

9. What part of an acre is 40 sq. rd.? 80 sq. rd.?

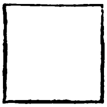
10. What part of a section is 80 A.? 160 A.? 320 A.?

79. A ***Surface*** has two dimensions—*length* and *breadth.*

80. A ***Rectangle*** is a plane figure bounded by four sides, having all its *angles right angles.*


RECTANGLE.

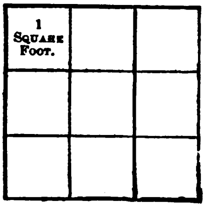
81. A ***Square*** is a rectangle bounded by four *equal* sides.


SQUARE.

1. How many square inches in a strip of board 1 inch wide and 12 inches long? In 2 such strips? In 3? In 4? In 5? In 6? 7? 8? 9? 10? 11? 12?

2. A *square yard* is a square, each side of which is 1 yd., or 3 ft. long.

This drawing represents a *square yard* divided into *square feet.*



3 sq. ft. × 3 = 9 sq. ft.

3. In 1 row there are 3 sq. ft., in 3 rows there are 3 times 3 sq. ft., or 9 sq. ft.

4. How many square feet in the surface of a table 3 feet wide and 6 feet long?

5. How many square feet in the floor of a room 8 ft. wide and 10 ft. long?

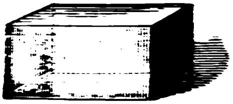
6. How many square feet in a hall 6 ft. wide, 12 ft. long? How many square yards? How many yards of carpet 1 yd. wide will cover it?

7. How do we find the *surface* or contents of a square or rectangular figure?

Ans. By taking the product of the numbers denoting its *two dimensions*—the length and breadth.

82. A ***Solid*** or ***Body*** has three dimensions—*length*, *breadth*, and *thickness*.

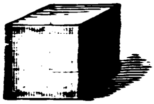
83. A ***Rectangular Solid*** is a body bounded by six rectangular plane *faces*.



The opposite sides are equal and parallel.

84. A ***Cube*** is a body bounded by six equal squares, or *faces*.

The sides of these squares are called the *edges* of the cube.



CUBE.

85. ***Cubic*** or ***Solid Measure*** is used in measuring *solids*; as timber, wood, stone, boxes of goods, etc.

TABLE.

1728 Cubic In. (*cu. in.*) = 1 Cubic Ft., *cu. ft.*
27 Cubic Feet = 1 Cubic Yd., *cu. yd.*

1 *cu. yd.* = { 46656 *cu. in.*
27 *cu. ft.*

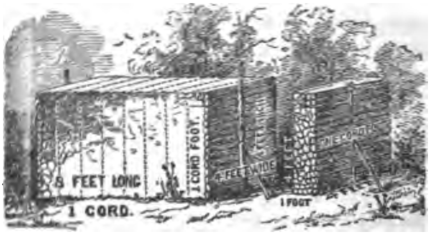
86. ***Wood Measure*** is used to measure wood and rough stone.

TABLE.

16 Cubic Feet = 1 Cord-Foot . . . . *cd. ft.*
8 Cord-Feet, or 128 Cubic Feet } = 1 Cord . . . . *Cd.*

A *Cord* of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

A *cord-foot* is 1 foot in length of such a pile; that is, 1 ft. long, 4 ft. wide, and 4 ft. high.

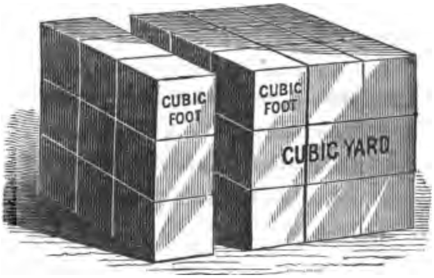


87. *1.* How many cubic feet in a piece of timber 1 foot square at the ends and 3 feet long? In 2 such pieces?

2. A *cubic yard* is a cube each *face* of which is 1 sq. yd., and each *edge* 1 yd., or 3 ft., long.

This drawing represents a *cubic yard*, each face being a *square* yard, containing 9 sq. ft.

If a piece, 1 ft. thick is cut from one side, it may be divided into 3 times 3 cu. ft., or 9 cu. ft. And since a cubic yard contains 3 such sections, there are 3 times 9 cu. ft., or 27 cu. ft., in a cubic yard.



9 cu. ft. × 3 = 27 cu. ft.

3. How many cubic feet in a block of marble 1 ft. thick, 3 ft. wide, and 6 ft. long? 2 ft. wide, 2 ft. thick, and 4 ft. long?

4. How many inches in a cubic block whose edges are 3 inches long? Are 4 inches long?

5. How do we find the *solidity*, or contents, of a cube or of an oblong body?

Ans. By taking the product of the numbers denoting the *three dimensions*—the length, breadth, and thickness.

6. How many cubic feet in a pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high? In a pile 6 ft. long, 5 ft. wide, and 3 ft. high?

7. If a stove burn 1 cord-foot of wood in a week, in what time will it burn 1 cord? 2 cords? 3 cords?

88. ***Capacity*** signifies extent of *room* or *space*.

89. Measures of capacity are divided into two classes: Measures of *Liquids* and Measures of *Dry Substances.*

90. ***Liquid Measure*** is used in measuring *liquids;* as spirituous liquors, oil, molasses, milk, water, etc.

TABLE.

4 Gills (*gi.*) = 1 Pint . . . *pt.*
2 Pints = 1 Quart . . *qt.*
4 Quarts = 1 Gallon . . *gal.*

1 *Gal.* = 32 *gi.*, 8 *pt.*, 4 *qt.*

1. How many gills in 1 pt.? In 2 pints? In 1 quart?

2. How many pints in 2 quarts? In 3 qt.? In 1 gal.?

3. How many pints in ½ gal.? In ¾ gal.? In 1½ gal.?

4. How many gallons in 12 quarts? In 24 qt.? In 36?

5. How many quarts in 10 pints? 18 pt.? In 20 pt.?

6. How many pints in 1 gal. 3 qt.? In 3 gal. 1 qt.?

7. What part of a gallon is 1 qt.? 3 qt.?

91. ***Dry Measure*** is used in measuring articles *not liquid;* as grain, fruit, salt, roots, etc.

TABLE.

2 Pints (*pt.*) = 1 Quart . . . *qt.*
8 Quarts = 1 Peck . . . *pk.*
4 Pecks = 1 Bushel . . *bu.*

1 *Bu.* = 64 *pt.*, 32 *qt.*, 4 *pk.*

8. How many quarts in 1 peck? In 3 pk.?

9. How many pints in 1 pk.? In 1 pk. 1 qt.?

10. How many quarts in ½ bushel? In ¾ bushel?

11. How many pecks in 2 bushels? In 1½ bu.? In 2¼ bu.?

12. How many half-pecks in a bushel? In 2 bu.?

13. In 48 quarts, how many pecks? How many bu.?

14. In 1 bu. 1 pk. 6 qt., how many quarts?

15. What part of a bushel is 8 qt.? 3 pk.?

92. The weight of the *bushel* of certain grains, seeds, and vegetables has been fixed in many of the States by law, and though *not uniform,* the following are the prevailing standards:

Wheat	60 lb.	Beans	60 lb.	Wheat Bran	20 lb.
Rye	56 "	Buckwheat	42 "	Rye Meal	56 "
Corn	56 "	Flax Seed	56 "	Corn Meal	50 "
Barley	48 "	Hemp Seed	44 "	Corn in Ear	68 "
Oats	32 "	Potatoes	60 "	Clover Seed	60 "
Peas	60 "	Onions	57 "	Timothy Seed	45 "

In the State of Illinois, a bushel of shelled corn is 56 lb.; of corn in the ear, 70 lb.; of corn meal, 48 lb.; of rye, 54 lb., and of mineral coal, 80 lb.

1. How many pounds in a half-bushel of wheat? In ½ bu. of corn? In ½ bu. of oats?

2. How many pounds in a peck of oats? Of barley?

3. At 4 cents a pint, what will 2 quarts of milk cost?

4. If a cup hold 3 pints, how many times can you fill it from a 3 gallon jar full of water?

5. If a bushel of plums cost 2 dollars, what is the cost of a peck? Of a quart?

6. How many quart boxes will 3 pk. 6 qt. of berries fill? How many half-gallon measures?

7. What is the cost of a peck of potatoes, at a cent a pound?

8. What is the cost of 5 bushels of corn in the ear, at ½ cent a pound?

9. At $2 a bushel, what must be paid for 12 bags of wheat, each weighing 120 pounds?

10. What is the cost of 600 pounds of wheat, at $1.50 a bushel?

93. ***Weight,*** on the earth, is the measure of gravity.

94. ***Avoirdupois Weight*** is used for all the ordinary purposes of weighing.

TABLE.

16 Ounces (*oz.*) = 1 Pound . . . . *lb.*
100 Pounds = 1 Hundredweight. *cwt.*
20 cwt., or 2000 lb. = 1 Ton . . . . . *T.*

$$1\ T. = \begin{cases} 32000\ oz. \\ 2000\ lb. \\ 20\ cwt. \end{cases}$$

The following denominations are also used:

100 Pounds of Grain or Flour make 1 Cental.
100 Pounds of Dry Fish " 1 Quintal.
100 Pounds of Nails " 1 Cask or Keg.
196 Pounds of Flour " 1 Barrel.
200 Pounds of Pork or Beef " 1 Barrel.

1. How many ounces in 3 lb.? In 1 lb. 10 oz.?

2. How many pounds in 4 hundredweight? In 6 cwt.?

3. How many hundredweight in 300 lb.? In 500 lb.?

4. What part of a hundredweight are 50 pounds? Are 25 pounds? Are 75 pounds?

5. How many hundredweight in $\frac{1}{4}$ of a ton of hay? In $\frac{1}{2}$ of a ton? In $\frac{1}{5}$ of a ton?

6. How many pounds in $\frac{1}{2}$ barrel of pork? In $\frac{1}{2}$ barrel of beef? In $\frac{1}{2}$ barrel of flour?

7. At 9 cents a pound, what will a keg of nails cost?

8. At 2 cents a pound, what will a cental of flour cost?

9. How many barrels are 600 pounds of pork?

10. How many centals are 500 pounds of flour?

11. How many kegs are 800 pounds of nails?

12. How many barrels are 1000 pounds of beef?

13. How many pounds in 3 half-barrels of flour? In $2\frac{1}{2}$ kegs of nails?

95. ***Troy Weight*** is used in weighing gold, silver, and jewels, and in philosophical experiments.

TABLE.

24 Grains (*gr.*)	= 1 Pennyweight . *pwt.*	1 lb. =	5760 *gr.*
20 Pennyweights	= 1 Ounce . . . *oz.*		240 *pwt.*
12 Ounces	= 1 Pound . . . *lb.*		12 *oz.*

96. ***Apothecaries' Weight*** is used by apothecaries and physicians in compounding *dry* medicines.

20 Grains (*gr.* xx)	= 1 Scruple . *sc.* or ℈.	℔ 1 =	*gr.* 5760.
3 Scruples (℈iij)	= 1 Dram . *dr.*, or ʒ.		℈ 288.
8 Drams (ʒ viij)	= 1 Ounce . *oz.*, or ℥.		ʒ 96.
12 Ounces (℥ xij)	= 1 Pound . *lb.*, or ℔.		℥ 12.

1. How many ounces in 2 pounds of gold? In 3 lb.?

2. How many pennyweights in 2 ounces? Ounces in 60 pwt.? Pounds in 48 ounces?

3. How many drams in 1 ounce? In 3? In 4? In 6?

4. How many ounces in ½ a pound? In ¼? In ¾?

5. How many drams in ℈30? In ℈45?

6. How many pounds in 72 oz.? In 96 oz.? In 144 oz.?

7. How many ounces in 3½ lb.? In 2¾ lb.? In ⅔ lb.?

8. What part of a pound is 4 oz.? 6 oz.? 8 oz.? 9 oz.?

9. How many powders of 6 grains each, can be made of 1 dram of medicine?

10. How many ounces in 1 pound of gold chain? In 1 pound of medicine? In 1 pound of coffee?

11. What will a gold chain, weighing 1 oz. 12 pwt., cost at $1 a pennyweight?

12. What part of a pound Troy are 4 oz.? 6 oz.? 8 oz.?

97. ***Time*** is the measure of a portion of duration.

TABLE.

60 Seconds (*sec.*) = 1 Minute . . . *min.*
60 Minutes = 1 Hour . . . *hr.*
24 Hours = 1 Day . . . . *da.*
7 Days = 1 Week . . . *wk.*
365 Days = 1 Common Year. *yr*
366 Days = 1 Leap Year . . *yr.*

COMMON YEAR.

1 *Yr.* = { 525600 *min.* 8760 *hr.* 365 *da.*

Name the *months* in the year, and the number of *days* in each.

	MONTHS.		NO. DAYS.		MONTHS.		NO. DAYS.
1.	January,	*Jan.,*	*31*	7.	July,	*July,*	*31*
2.	February,	*Feb.,*	*28* or *29*	8.	August,	*Aug.,*	*31*
3.	March,	*Mar.,*	*31*	9.	September,	*Sept.,*	*30*
4.	April,	*Apr.,*	*30*	10.	October,	*Oct.,*	*31*
5.	May,	*May,*	*31*	11.	November,	*Nov.,*	*30*
6.	June,	*June,*	*30*	12.	December,	*Dec.,*	*31*

98. ***Circular,*** sometimes called ***Angular Measure,*** is used in measuring angles, arcs of circles, etc.

TABLE.

60 Seconds (″) = 1 Minute . . . ′
60 Minutes = 1 Degree . . . °
30 Degrees = 1 Sign . . . *S.*
12 Signs, or 360° = 1 Circle . . . *C.*

1 *C.* = { 1296000″. 21600′. 360°. 12 *S.*

1. How many days in 1 week? In 3 wk.? In 4 wk.?

2. How many hours in $\frac{1}{4}$ of a day? In $\frac{1}{2}$? In $\frac{3}{4}$?

3. How many minutes in $\frac{1}{5}$ of an hour? In $\frac{1}{3}$? In $\frac{1}{3}$?

4. How many days has a leap year?

5. How often does leap year occur?

6. Name the months that have 30 da. each? 31 da.

7. What month is it now? How many days has it?

8. How many days from April 20th to May 10th?

9. How many days from Aug. 1st to Sept. 5th?

99. In market certain articles are bought and sold by *counting.*

TABLE.

12 Units = 1 Dozen . . . *doz.*
12 Dozen = 1 Gross . . . *gro.*
12 Gross = 1 Great Gross . *G. gro.*
20 Units = 1 Score . . . *sc.*

1 *G. gro.* = 1728 *units.* 144 *doz.* 12 *gro.*

***Two* things of a kind are often called a *pair,* and *six* things a *set;* as a *pair* of horses, a *set* of chairs, etc.**

The *paper* trade use the following:

24 Sheets = 1 Quire . . . . *qr.*
20 Quires = 1 Ream . . . . *rm.*
2 Reams = 1 Bundle . . . *bun.*
5 Bundles = 1 Bale . . . . *B.*

1 *B.* = 4800 Sheets. 200 Quires. 10 Reams. 5 Bundles.

1. How many buttons in 1 gross? In ½ of a gross?

2. How many pens in ¼ of a gross? How many dozen?

3. Find the cost of ½ of a gross of eggs, at 20 cents a dozen.

4. How many make 1 pair? 2½ pairs? 5 pairs?

5. How many in 1 set? In ½ a set? In 2½ sets?

6. How many quires of paper in ½ of a ream?

7. How many sheets of paper in 2 quires? In ½ of a quire?

8. How many score in 40? In 60? In 100?

9. How many in ½ of a score? In 1½ score? 2½ score?

10. Which is greater, ¼ of a ream or 100 sheets of paper?

11. What is the difference between ½ of a dozen, and 6 dozen eggs?

12. Which is greater, 6 sets of chairs, or 2½ dozen chairs?

100. *1.* What is the difference in a foot long and a foot square? In a square foot and a cubic foot?

2. How many inches long is a block in the form of a cubic foot? How many wide?

3. What is measured by the fathom? By the hand?

4. How many feet deep is a river that measures 5 fathoms?

5. How many feet high is a horse that measures 15 hands?

6. How many square feet in a blackboard 27 ft. long and 3 ft. wide? How many square yards?

7. Which is greater, 10 square inches or a 10 inch square? What is the difference?

8. Which is greater, 6 cubic inches or a 6 inch cube?

9. How many squares, each equal to a square foot, are equal to a surface 12 feet long and 8 feet wide?

10. How many cubes, each equal to a cubic foot, are equal to a block 5 ft. long, 4 ft. wide, and 3 ft. thick?

11. How is stone measured? How is wood measured?

12. Give the dimensions of a cord-foot.

13. How many pints of water will fill a vessel that holds 1½ gallons?

14. How many times can a peck measure be filled from 2½ bushels?

15. Which is heavier, a bushel of wheat or a bushel of corn? A bushel of barley or a bushel of oats?

16. Which is heavier, an ounce of calomel or an ounce of gold? A pound of tea or a pound of silver?

17. Which is heavier, a barrel of flour or a barrel of pork? A keg of nails or a cental of grain?

101. *1.* How much must be paid for 12 weeks' board, at $.75 a day?

2. How much must be paid for 128 pints of chestnuts at $3.50 a bushel?

3. Paid $17 for a barrel of beef. What is that a pound?

4. How many square feet of oil-cloth will cover an office floor 15 ft. long and 12 ft. wide? How many square yards?

5. What will half a quintal of fish cost, at 6 cents a pound?

6. What will a ton of hay cost, at half a cent a pound?

7. Bought a peck of peanuts for $.80; sold them at 6 cents a pint. What was the gain?

8. How many bushels of corn-meal can be bought for $30, at 2 cents a pound?

9. How many acres in 25600 square rods?

10. How many sections in 1920 acres?

11. Change 2 reams 10 quires to sheets.

12. In 40 score how many single things?

13. In 14 great gross how many dozens?

14. In 120 gross how many score?

15. In the 3 Autumn months how many hours?

16. In the three Summer months how many minutes?

17. In 75 cords how many cubic feet?

18. How many square rods in a field 90 rods long and 75 rods wide? How many acres?

19. A pond of water measures 3 fathoms 2 feet 9 inches in depth; how many inches deep is it?

20. An innkeeper bought a load of 40 bushels of oats, at 36 cents a bushel, and retailed them at 25 cents a peck; how much did he make on the load?

21. In a township of land, how many sections? How many acres?

22. What will be the cost of 3 miles of telegraph wire at 3 cents a foot?

23. What is the difference in feet, between the length and breadth of your school-room?

24. A man bought a piece of cloth containing 45 yards and sold 69 feet. How many yards had he left?

25. How many bushels in 1000 pounds of wheat? Of rye? Of corn? Of oats? Of beans? Of corn-meal? Of barley? Of buckwheat? Of wheat bran?

26. John travels 20 miles east, and Reuben 65 miles west. How far apart are they? How far apart would they have been if they had both traveled east?

27. If a man walk 4 miles an hour, and 10 hours a day, how many miles can he walk in 24 days?

28. How many quarts will a bushel, a half-bushel and a peck measure all contain?

29. What is the age of a man 3 score and 5 years old?

30. How much will I receive for a load of wheat weighing 2760 pounds at $1.50 per bushel?

31. How many cubic feet in a stick of timber 32 feet long, 2 feet wide and 1 foot thick?

32. In 3 tons of cheese how many pounds?

33. How much will 4 cheese cost, each weighing 36 pounds, at 9 cents a pound?

34. What would be the value of a city lot 40 feet wide and 120 feet long, at 2 cents a square foot?

35. In a board 12 feet long and 2 feet wide, how many square feet?

36. How long is the top of your desk? How wide? How far is it around it? How many square inches does it contain? How many square feet?

37. In a block of marble 6 feet long and 3 feet square, how many cubic feet?

38. In a pile of wood 26 feet long 6 feet high and 3 feet wide, how many cubic feet? How many cords?

39. How many square feet in the blackboard?

40. Measure five rectangular surfaces and compute their square contents.

41. What is the cost of a block of marble, 4 ft. long 4 ft. wide and 3 ft. thick, at $2 a cubic foot?

42. Measure four rectangular solids and compute their contents.

43. How long and wide is a pane of glass in the school-house window? How many panes in all the windows in the school-room? How many square inches in each pane?

44. How many square feet of glass are there in one window? How many in all the windows in the school-room?

45. How many bushels of oats in a load weighing 1280 pounds?

46. How many bushels of wheat in a load weighing 2175 pounds?

47. A grocer bought 3 barrels of flour at $6 a barrel, and sold it out at 4 cents a pound; what did he gain on the whole?

48. Bought 1 T. 15 cwt. 36 lb. of sugar at 7 cents a pound; what did it cost?

49. How many rods of fence will inclose a farm a mile square?

50. What will a peck of clover-seed cost, at 20 cents a lb.?

51. What is the cost of a load of oats weighing 1860 pounds, at $.56 a bushel?

52. How many cubic feet in a room 18 feet long, 16 feet wide, and 10 feet high?

53. From a *section* of land, there was sold 240 acres; how much remained?

54. How many bushels of wheat in 48 sacks, each containing 165 pounds, and what is it worth at $1.50 a bushel?

55. How much will it cost to dig a cellar 24 ft. long, 18 ft. wide and 6 feet deep, at 1 cent a cubic foot?

56. If granite weigh 175 pounds a cubic foot, what is the weight of a cubic yard?

57. A grocer bought 4 barrels of cider, each containing 36 gallons, at $2 a barrel, and after converting it into vinegar, he retailed it at 15 cents a gallon; what was his whole gain?

58. School continues 200 days in a year, 6 hours each day; how many minutes does a pupil attend, who is neither absent nor tardy?

59. Bought 200 barrels of flour for $1850; sold 86 barrels of it at $10 a barrel, and the remainder at $12 a barrel. Did I gain or lose; and how much?

60. What will a ton of corn-meal cost, at $1.20 a bu.?

61. An Illinois farmer sold a load of corn weighing 2496 lb., and a load of oats weighing 1920 lb.; for the corn he received $.62 a bushel, and for the oats $.44 a bushel. What did he receive for both loads?

ELEMENTS OF ARITHMETIC, PART II.

102. The work in the following pages, being designed for the next *higher grade* of pupils, consists mainly of a review and enlargement of the subjects treated in the foregoing part of this book.

For the purposes of drill and fuller review, frequent reference should be made to the *oral work* and *drill exercises* of the preceding grade.

Several new subjects are also presented in their elementary principles and applications.

PRELIMINARY DEFINITIONS.

103. Arithmetic is the Science of Numbers, and the Art of Computation.

As a science, Arithmetic treats of the nature and properties of numbers. As an art, it teaches how to apply a knowledge of numbers to practical and business purposes.

104. A ***Unit*** is a single thing; as, *one*, *one* ten, etc. (**1.**)

105. A ***Number*** is a unit, or collection of units, and represents whatever answers to the question, "*How many?*" as, *one* boy, *four* books, *six* miles.

106. An ***Integral Number*** or ***Integer*** is a number which expresses whole things; as *seven*, *four* days.

107. The ***Unit of a Number*** is *one* of the collection of units which constitute the number. Thus, the unit of five is *one*, of eight boys is *one* boy, of ten cents is *one* cent.

108. A ***Concrete Number*** is a number that is applied to a particular kind of object, or quantity; as three *houses*, four *dollars*, five *minutes*.

109. An ***Abstract Number*** is a number that is not applied to any object; as *four, seven, eight.*

110. ***Like Numbers*** are such as have the same kind of unit, or express the same kind of quantity. They may be either concrete or abstract; as *eight* and *nine,* six *days* and ten *days,* two *rods* five *feet* and five *rods* three *feet.*

111. ***Unlike Numbers*** are such as have different kinds of units, or express different kinds of quantity; as ten *months* and eight *miles;* seven *dollars* and five *barrels.*

112. A ***Scale*** in Arithmetic is a succession of units, increasing and decreasing according to a certain law, or rule. Scales are *uniform* or *varying.*

113. A ***Uniform Scale*** is one in which the law of increase and decrease is the same throughout the entire succession of units.

114. A ***Varying Scale*** is one in which the law of increase and decrease is not the same throughout the entire succession of units.

115. A ***Decimal Scale*** is one in which the law of increase and decrease is *uniformly* TEN.

NOTATION AND NUMERATION.

116. ***Notation*** is a method of *writing,* or representing numbers by characters.

117. ***Numeration*** is a method of *reading* numbers represented by characters.

118. ***Figures*** are characters used to express numbers.

119. The ***Arabic Notation*** is the method of expressing numbers by *figures.* It is so called, because it is said to have been invented by the Arabs.

120. This method employs *ten* different characters, or figures, to represent numbers, viz.:

Figures.	0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Names.	Naught,	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine.

121. The first character, or *cipher,* is called *Naught,* or *Zero,* and when standing alone has no value.

The other nine are called *significant* figures, because each has a value of its own. They are also called *digits.*

These ten characters, when combined according to certain principles, can be made to express any number.

The *first nine* numbers are each represented by a *single figure,* and are called *units* of the *first order.*

122. By grouping ten ones, or *units* of the *first* order into a larger collection, there is formed a unit of the *second* order, called *ten,* which is represented by writing the figure 1 with a cipher after it; thus, 10.

In the same manner are represented *two* tens, or twenty, by 20; *three* tens, or thirty, by 30; *four* tens, or forty, by 40, etc. The numbers between *ten* and *twenty, twenty* and *thirty, thirty* and *forty,* to *ninety-nine,* are represented by writing the significant figures 1, 2, 3, 4, etc., in the units or *first* place. Thus, eleven is expressed by 11, twelve by 12, thirteen by 13, twenty-four by 24, thirty-five by 35, etc.

123. By grouping ten units of the *second order,* or *ten tens,* into a larger collection, there is formed a unit of the *third* order, called a *hundred,* represented by writing the figure 1 with two ciphers after it; thus, 100.

In the same manner are represented *two* hundred, by 200; *three* hundred, by 300; *four* hundred, by 400, etc.

The numbers between one hundred and nine hundred ninety-nine are represented by writing the *tens*, in the *second* place, and the *units*, in the *first* place. Thus, two hundred twenty-five is expressed by 225; four hundred thirty-seven, by 437, etc.

124. ***Orders of Units*** are denoted by the *position* of the figures used in expressing a number. (4)

Thus, 532 represents 2 units of the *first order*, 3 units of the *second order*, or 3 *tens*, and 5 units of the *third order*, or 5 *hundreds*, and is read *five hundred thirty-two.*

125. PRINCIPLES.—*1. Ten units of any order in a number make one unit of the next higher order.*

2. When any order of units in a number is vacant, the place is filled with a cipher.

From the preceding illustrations it is obvious, that,

126. Moving a figure one place to the *left*, *increases* its *representative* value *tenfold;* and,

127. Moving a figure one place to the *right*, *diminishes* its *representative* value *tenfold.*

128. Copy and read the following, and name the number of *hundreds, tens,* and *units* in each.

(*1.*)	(*2.*)	(*3.*)	(*4.*)	(*5.*)	(*6.*)
231	128	727	810	190	804
426	561	636	244	927	792
147	600	235	506	879	916
320	284	317	789	586	687

129. This method of numeration groups the successive orders into ***periods*** of *three* figures each. The periods are commonly separated by commas, each period taking the name of its lowest order, as shown in the following

NUMERATION TABLE.

PERIODS.	6th.			5th.			4th.			3d.			2d.			1st.		
NAME.	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
ORDERS OF UNITS IN THE PERIODS.	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
NUMBER.		3	0,	2	9	1,	0	4	0,	0	2	7,	3	0	6,	4	0	0

The number is read 30 *quadrillion,* 291 *trillion,* 40 *billion,* 27 *million,* 306 *thousand,* 400.

1. In *reading* numbers, the name of the units period is omitted.
2. Every period except the highest must contain *three* figures.

130. RULE FOR NOTATION.—*Begin at the left, and write the hundreds, tens, and units of each period in their proper order, filling all vacant places and periods with ciphers.*

131. RULE FOR NUMERATION.—I. *Begin at the right and separate the number into periods of three figures each.*

II.—*Begin at the left, and read each period as if it were units, giving its name.*

EXERCISES IN NOTATION AND NUMERATION.

132. Write in figures and read:

1. Six units of the 3d order, five of the 2d, and four of the 1st.

2. Five units of the 4th order, seven of the 2d, and six of the 1st.

3. Eight units of the 4th order and four of the 2d.

4. Five units of the 4th order and eight of the 2d.

5. Three units of the 5th order, six of the 4th, four of the 3d, and seven of the 1st.

Write and read the following numbers in figures:

6. Twenty-five units in the 2d period, and four hundred ninety-six in the 1st.

7. Four hundred thirty-six units in the 4th period, twelve in the 3d, one hundred in the 2d, and three hundred and one in the 1st.

Express the following numbers by figures:

8. Twenty-six thousand twenty-six.

9. Fourteen thousand two hundred eighty.

10. One hundred seventy-six thousand.

11. Four hundred fifty thousand thirty-nine.

12. Seven million thirty-six.

13. Five hundred sixty-three thousand four.

14. One million ninety-six thousand.

15. Ten million ten thousand ten hundred ten.

Point off and read the following numbers:

16. 24835.
17. 2474783.
18. 31628045.
19. 247843112.
20. 100103.
21. 53000008.
22. 406270035.
23. 3730016000.

DRILL EXERCISES.

133. *1.* Make a table showing the orders used in expressing numbers up to the 7th order (**Art. 5**).

2. Write ten numbers, each containing seven *orders*, and practice naming the orders until the orders up to and including *millions* are perfectly familiar.

3. *7 7 7 7 7 7 7.*

Practice on the above by saying, Seven units; seven tens, or seventy, which is ten times seven units; seven hundreds, or seven hundred, which is ten times seven tens; seven thousands, or seven thousand, which is ten times seven hundred, etc.

4. Reverse the process, thus: Seven million; seven hundreds of thousands, or seven hundred thousand, which is one-tenth of seven million; seven tens of thousands, or seventy thousand, which is one-tenth of seven hundred thousand, etc.

Practice the last two exercises until the uniform scale of increase or decrease is well understood.

5. Let the pupil write several numbers, each consisting of six *periods*. Thus,

333,333,333,333,333,333.
500,500,500,500,500,500.

Drill on these in the same manner as in the orders, to fix in the mind that each period represents a value one *thousand* times as large as the period on its right, and one-*thousandth* as large as the period on its left.

134. DRILL TABLE No. 2.

	A.	B.	C.	D.	E.	F.	G.
12.	7	10	12	6	70	500	6000
11.	11	8	4	8	80	400	4000
10.	2	11	6	7	60	700	11000
9.	10	7	1	10	20	800	10000
8.	8	4	3	5	70	1200	7000
7.	1	12	9	1	30	1000	3000
6.	12	1	10	3	100	1200	1000
5.	9	6	2	12	60	100	12000
4.	3	3	7	9	90	1000	6000
3.	5	5	5	2	110	600	4000
2.	4	9	11	4	120	300	12000
1.	6	2	8	11	50	200	8000

The above table is to be used in a way very similar to the table of drill exercises on Page 53. For further directions consult the specific notes on that page.

The teacher will observe that besides the drill in *Addition*, *Subtraction*, *Multiplication*, and *Division*, this and similar tables also afford abundant exercise in *Numeration*. The exercises of *adding*, etc., *at sight*, also involve *numerating at sight*. Further exercise may be had by rapidly reading the columns and lines. Also, by mentally prefixing one or more figures to each number and then reading rapidly by column and by line.

ADDITION.

135. ***Addition*** is the process of finding a number equivalent to two or more numbers.

136. The ***Sum*** or ***Amount*** is the number obtained by addition.

For definitions, etc., of the *signs* + and =, see Page 11.
For definitions, etc., of the *signs* $ and (.), see Page 65.

137. An ***Equation*** is an expression of *equality* between two numbers or sets of numbers.

All that is written *before* the sign of *equality* is called the *first member* of the equation; all that is written *after* the sign of equality is called the *second member.*

The numbers in each member are called the *terms* of the equation.

Thus, $6+4=10$ is an *equation,* and is read 6 *plus* 4 *equals* 10, and means that the sum of 6 and 4 is equal to 10. $6+4$ is the *first member* of the equation, and 10 is the *second member;* and 6, 4, and 10 are the *terms* of the equation.

Name the *members* and the *terms* of each of the following equations:

1. $9+12=21$
2. $14+10=24$
3. $26+9=35$
4. $44+12=56$
5. $14+40=44+10$
6. $18+20=31+7$

138. PRINCIPLE.—*Only like numbers and units of the same order can be added.*

ORAL EXERCISES.

139. Let the teacher read the *first* member of the equation, and the pupil promptly give the *second* member.

The expression "=?" is read *equals how many*, or *what.*

9+ 8+ 5=?
8+ 5+ 9=?
10+ 7+ 6=?
6+10+ 7=?
11+ 5+ 8=?
15+ 3+10=?

20+ 7+3=?
11+ 9+1=?
12+ 7+9=?
21+10+7=?
24+ 6+5=?
10+25+9=?

4+15+10=?
9+ 9+ 9=?
23+10+ 6=?
7+21+11=?
14+ 6+12=?
26+10+ 9=?

140. Read the following equations:

1. $12.+$8.=$20.
2. $25.+$10.=$35.
3. $3.25+$6.75=$10.
4. $.75+$.20=$.95.
5. $.60+$.40=$1.00.
6. $14.08+$3.14=$17.22.

141. Express the following by proper figures and signs:

1. Nine dollars and thirty cents.
2. Thirty dollars and ten cents.
3. Eighty-four cents.
4. Seventy-eight cents.
5. Six dollars and sixteen cents.
6. 7 dollars and 26 cents.
7. 9 dollars and 5 cents.
8. 19 dollars and 7 cents.
9. 69 cents; 23 cents.
10. 10 cents; 6 cents.

The teacher may exercise the class *orally*, by dictating rapidly, but distinctly, similar examples. Thus, *Sign, five, three?* The prompt response should be, "Fifty-three dollars" ($53). *Ques. Sign, point, seven, four?* *Ans.* Seventy-four cents ($.74). *Ques. Sign, point, naught, eight?* *Ans.* Eight cents ($.08), etc.

Also, the converse; thus, *Ques.* "Forty-five dollars" ($45)? *Ans. Sign, four, five.* *Ques.* Fifty-six cents ($.56)? *Ans. Sign, point, five, six.* *Ques.* Nine dollars seven cents ($9.07)? *Ans. Sign, nine, point, naught, seven;* etc.

142. In arranging for addition, dollars should be written under dollars, and cents under cents, in such order that the *points* stand in a *vertical* line. (72.)

The *sign* $, and the *point* (.) should never be omitted.

For further Oral and Drill work, review Pages 14 to 19.

WRITTEN EXERCISES.

1. What is the sum of 467, 536, 84, and 705 ?

OPERATION.

```
 467
 536
  84
 705
1792 Sum.
```

ANALYSIS.—Arranging the numbers as before, begin at the right hand and add the column of units; thus, 5, 9, 15, 22 *units*, equal to 2 tens and 2 units. Write the 2 units in the units' place, and reserve the 2 tens to add to the next column.

Next, adding the 2 tens reserved, to the column of tens, say, 2, 10, 13, 19 *tens*, equal to 1 hundred and 9 tens. Write the 9 tens in the tens' place, and reserve the 1 hundred to add to the next column.

Lastly, adding the 1 hundred reserved, say, 1, 8, 13, 17 *hundreds*, equal to 1 thousand, and 7 hundreds, which write in hundreds' and thousands' places. Hence the *sum* is 1792.

PROOF.—Add the columns in the reversed direction. If the two results agree, the work is probably correct.

In like manner, copy, add, and prove,

(*2.*)	(*3.*)	(*4.*)	(*5.*)
276 miles.	876 feet.	$20.30	$145.24
307 "	94 "	7.56	36.60
638 "	142 "	13.08	105.08
425 "	507 "	25.	.75

6. Find the sum of $370.21, $2.49, $3.07, and $.94.

7. Find the sum of 2008, 1400, 706, 300, and 77.

8. If 4 loads of coal weigh respectively 1922, 1609, 2100, and 1873 pounds, what is the entire weight ?

RULE.—I. *Write the numbers so that figures of the same order stand in the same column.*

II. *Beginning at the right, add each column separately, and write the sum, if expressed by one figure, under the column added.*

III. *If the sum of any column consists of two or more figures, write the unit figure under that column, and add the remaining figure or figures to the next column.*

PROOF.—*Add each column in the reverse direction. If the results agree, the work is correct.*

Copy and add the following:

9. 627, 826, and 679.
10. 574, 786, and 895.
11. 1284, 96, and 706.
12. 102, 2406, and 500.
13. 3147, 7605, and 8297.
14. 6078, 9578, and 5809.
15. 7100, 873, and 36086.
16. 609, 8004, and 50326.

(*17.*)	(*18.*)	(*19.*)	(*20.*)	(*21.*)
20992	38912	$354.90	639	543678
5248	9728	64.57	91758	2766
1312	2432	946.07	9347	28345
328	304	64.63	81731	563875
41984	1216	346.	9342	7427

22. 123 + 456 + 785 + 12 + 345 + 901 + 567 + 48 = ?
23. 12345 + 67890 + 8763 + 347 + 1037 + 198760 = ?
24. 172 + 4005 + 3761 + 20472 + 367012 + 19762 = ?
25. $246.26 + $85.50 + $100.09 + $17.75 + $.96 = ?
26. $84.90 + $760 + $1235.18 + $56.73 + $1000 = ?
27. $4500 + $63.19 + $508.07 + $24.30 + $300.05 = ?

28. The Duke of Wellington's army at Waterloo consisted of 26661 infantry, 8735 cavalry, 6877 artillery, and 33413 allies. What was the whole number of his army?

29. Napoleon's army at Waterloo was composed of infantry 48950, cavalry 15765, and artillery 7732. What was the whole number of his army?

30. Gave $325 for a horse, $275.50 for a carriage, $75.75 for a harness, and $20.62 for a robe. What was the cost of the whole?

31. Bought a pair of boots for $8.50, an umbrella for $3.62, a pair of gloves for $1.25, some collars for $.75, and a hat for $4. What was the whole cost?

32. 17853 + 85471 + 92846 + 15698 + 87659 + 1932881 = ?

33. 76543 + 87884 + 99772 + 88594 + 66777 + 8778896 = ?

34. $3675 + $40706 + $2008 + $540 + $84 + $60508 = ?

35. 4703 + 60173 + 345 + 18040 + 20009 + 8090 feet = ?

36. A flour merchant sold 1728 barrels of flour one month, 2009 the next, and as many the third as in the other two. How many barrels did he sell in three months?

37. A father divided his estate among his four sons and two daughters, giving each son $3896, and each daughter $2675. What was the value of his estate?

38. A man engaging in trade, gained $514.50 the first year, $1084 the second, and as much the third as he gained during the first and second. What was his whole gain?

39. Bought three village lots for $12750, and sold them so as to gain $745.50 on each lot. For what were they sold?

40. A has $2248, B has $3677, and C has 800 more than A and B together. How many dollars have all?

41. Bought a horse for $154, and sold him for $35.75 more than he cost. For how much did I sell him?

42. A lady paid 12 dollars for a scarf, 3 dollars and 25 cents for a fan, two dollars for a pair of gloves, and 42 cents for a collar. How much did she pay for all?

43. A lady gave $48.50 for silk for a dress, $16.75 for the trimmings, and $15.62 for making. What was the cost of the dress?

(*44.*)	(*45.*)	(*46.*)	(*47.*)	(*48.*)	(*49.*)
$99.84	96256	$117.76	98304	1728	$675.84
24.96	6016	29.44	6144	864	168.86
6.24	376	7.36	384	108	10.56
1.56	141	1.84	24576	81	1.32
12.48	188	6.68	3072	5296	.96
.98	1504	58.88	144	3456	2.64
3.12	752	1.38	49152	432	84.48

50. A man gave his property to his wife, two sons, and four daughters. To his daughters he gave $4370 each, to his sons $6045 each, and to his wife as much as he gave to one son and one daughter. What was the value of his entire property?

(*51.*)	(*52.*)	(*53.*)	(*54.*)
8786	78982	437987	3216677
5968	69771	566789	4569911
8789	68339	544321	6543344
9896	56234	891389	9576677
3674	95876	219721	1539900
1767	57412	625247	6662233
5989	73375	431321	4235566

55. How many miles from the southern extremity of Lake Michigan to the Gulf of St. Lawrence, passing through Lake Michigan, 330 miles; Lake Huron, 260 miles; River St. Clair, 24 miles; Lake St. Clair, 20 miles; Detroit River, 23 miles; Lake Erie, 260 miles; Niagara River, 34 miles; Lake Ontario, 180 miles; and the River St. Lawrence, 750 miles?

56. The estimated area of Lake Superior is 31400 square miles; Huron, 23800; Michigan, 25600; Great Bear, 9300; Great Slave, 12000; Winnipeg, 8900; Erie, 10000; Ontario, 7300. What is the area or surface of all these lakes?

57. What is the sum of 5736 dollars and 45 cents, 1000 dollars and 80 cents, 405 dollars and 15 cents, 50 dollars and 9 cents, and 79 cents?

58. Find the sum of twenty-five hundred dollars, 420 dollars and 47 cents, $23 and fifty cents, $600, and ten dollars and eight cents.

59. 1 million 400 thousand and 50 + 15 hundred + 25 thousand + 120 thousand 6 hundred and 14 = ?

60. Paid $3456 for a house, $426.75 for painting it, $2809.48 for the furniture. What was the cost of the whole?

61. North America has an area of 8825537 square miles, South America 6954131 square miles, and the West Indies 93810 square miles. What is the area of the entire American continent?

62. A man owns farms valued at $62500, city lots worth $10260, a house worth $21300, and other property to the amount of $10500. What is the total value of his property?

A great variety of examples may be made from the following *Ledger Columns* by copying and adding the numbers in the *first line* of the *four* columns, then, the *second line*, etc., until all have been used in the same manner.

Then copy and add *four* numbers in *first* column, then *five* numbers, then *six*, and so on up to *ten* numbers. Do the same with all the columns.

(63.)	*(64.)*	*(65.)*	*(66.)*
$ 42.17	$ 506.76	$2371.67	$14763.84
36.24	194.32	4571.84	33276.90
18.42	427.90	1690.50	47061.39
10.71	173.26	2037.69	18242.76
194.30	71.32	5094.46	37364.96
347.16	39.46	876.54	8410.31
40.00	152.60	679.81	5724.27
12.94	271.78	155.48	56317.66
86.73	320.00	4930.71	81742.73
271.19	709.08	3104.13	22431.27
103.07	48.50	1987.67	40163.55
500.50	63.41	5142.84	32189.60
7.59	56.00	276.30	7063.21
11.44	410.10	522.71	3451.09
81.92	372.22	3114.60	9200.00
110.10	137.89	1776.82	1807.36
107.09	276.44	7152.91	56768.72
207.16	18.19	9328.42	63024.27
97.20	27.96	472.19	36180.45
21.77	157.16	321.42	90807.08
150.15	94.57	2423.79	28763.81
427.26	177.66	1600.81	37196.75
316.42	327.40	5976.27	4230.61

SUBTRACTION.

143. ***Subtraction*** is the process of finding the difference between two numbers.

144. The ***Minuend*** is the greater of the two numbers compared to find their difference.

145. The ***Subtrahend*** is the smaller of the two numbers.

146. The ***Difference*** or ***Remainder*** is the result obtained by subtracting. **(16.)**

For the *sign* of subtraction, etc., see Page 21.

147. ***Parentheses*** ( ) denote that the numbers within them are to be considered together. (Page 36.)

A ***Vinculum*** $\overline{\qquad}$ has the same signification. Thus, 15 + (12 — 7), or 15 + $\overline{12 - 7}$, signifies that to 15 is to be added the difference between 12 and 7.

148. Principles.—*1. Only like numbers and units of the same order can be subtracted.*

2. The minuend must be equal to the sum of the subtrahend and remainder.

What is the difference between

17 and 4+6 ?
24 and 9+5 ?
18 and 7+7 ?
20 and 6+6 ?
35 and 9+20 ?
28 and 9+ 9 ?
12+5 and 24+3 ?
27—8 and 14+8 ?
30—10 and 9+ 8 ?
36+ 9 and 50—25 ?
42—12 and 30—10 ?
55+ 5 and 70—10 ?

Similar dictation exercises may be given by the teacher.

ORAL EXERCISES.

149. *1.* From a cask containing 52 gallons, 27 gallons were drawn out. How many gallons remained?

ANALYSIS.—The difference between 52 gallons and 27 gallons. 27 is 2 tens and 7 units. 2 tens or 20 from 52 leaves 32, and 7 from 32 leaves 25. Hence 25 gallons remained in the cask.

2. From a piece of cloth containing 46 yards, 24 yards were cut. How many yards were left?

3. A man bought a watch for $40, and a chain for $15, and sold both for $63. How much did he gain?

4. How many are 6 and 40, less 5 and 20?

5. How many are 7 and 30, taken from 5 and 50?

6. Eighteen plus 12 equals 40 minus how many?

7. Twenty-two plus 15 equals how many plus 10?

8. William having 75 cents, gave 25 cents for a book and 20 cents for a slate. How many cents had he left?

9. A farmer sold a horse for $96, which was $23 more than the horse cost. What did he cost?

Find the *omitted* term in the following equations:

10. $12+8-6=?$

11. $46+12-14=?$

12. $57-13+8=?$

13. $60-24+6=?$

14. $28+6=40-?$

15. $42-12=18+?$

16. $30+25=?+40$

17. $27-11=19-?$

18. $36+16=60-?$

19. $54-12=?+12$

20. $17+23=56-?$

21. $18+25=23+?$

22. $64-48=30-?$

23. $75-30=?+15$

24. $16+38=60-?$

25. $43+?=27+28$

26. $80-?=100-40$

27. $22+54=64+?$

For Oral and Drill Exercises review Page 26.

WRITTEN EXERCISES.

150. *1.* From 953 subtract 674.

OPERATION.

	8 14 13
Minuend,	9 5 3
Subtrahend,	6 7 4
Remainder,	2 7 9

ANALYSIS.—Write the less number under the greater, so that units of the same order stand in the same column.

Since 4 units cannot be subtracted from 3 units, increase the 3 units by a unit from the next higher order, or 10 units, making 13 units. 4 units from 13 units leave 9 units, which write in the units' place.

Since 1 of the tens was united with the units, there are 4 tens left. As 7 tens cannot be subtracted from 4 tens, increase the 4 tens by a unit from the next higher order, or 10 tens, making 14 tens. 7 tens from 14 tens leave 7 tens, which write in the tens' place.

Since 1 of the hundreds was united with the tens, there are 8 hundreds left. 6 hundreds from 8 hundreds leave 2 hundreds, which write in the hundreds' place. Hence the remainder is 279.

PROOF.—The remainder 279 added to the subtrahend 674, equals 953 the minuend. Hence the work is correct.

	(*2.*)	(*3.*)	(*4.*)
From	42670 miles.	51062 acres.	246700 feet.
Take	14384 "	24300 "	18030 "

When one of the given numbers contains *cents*, and the other does not, fill the vacant places with two ciphers.

	(*5.*)	(*6.*)	(*7.*)	(*8.*)
From	$325.17	$279.00	$105.08	$7.00
Take	84.36	183.42	67.00	.84

What is the difference between

9. 34726 and 47062?

10. 57600 and 20012?

11. 70361 and 1005?

12. 7620 and 12420?

13. $4027 and $703.41?

14. $1076 and $2340.50?

Rule.—I. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

II. *Begin at the right, and subtract the units of each order of the subtrahend from the units of the corresponding order of the minuend, and write the result beneath.*

III. *If the units of any order of the subtrahend are greater than the units of the corresponding order of the minuend, increase the latter by* 10, *and subtract; then diminish by* 1 *the units of the next higher order in the minuend, and proceed as before.*

Proof.—*Add the remainder to the subtrahend, and if the sum is equal to the minuend, the work is correct.*

Instead of *diminishing* by 1 the units of the next higher order in the *minuend,* we may *increase* by 1 the units of the next higher order in the *subtrahend.*

Subtract

15. 20762 from 53120.
16. $73.16 from $138.
17. $247 from $382.28.

From

18. $430.09, take $272.46.
19. 15200 rods, take 6472 rods.
20. 120764 tons, take 75028 tons.

How many years from the date of each of the following events to the present year?

21. Figures were used by the Arabs in the year 890.

22. Decimal fractions were invented in 1464.

23. Printing was invented in 1441.

24. The telescope was invented by Galileo in 1610.

25. The electric telegraph was first used in the United States in 1844.

26. The first passage of the Atlantic Ocean by steam was in 1839.

27. America was discovered in 1492.

28. American Independence was declared in 1776.

29. George Washington was born in 1732.

30. First steamboat was built by Robert Fulton in 1808.

31. Cotton was first planted in the United States in 1769.

32. Newspapers were first published in 1630.

33. Glass windows first used in England in 1180.

34. The great fire in Chicago was in 1871.

Find the difference between

35. 1204 and 5379.

36. 1370 and 1421.

37. 8673 and 3560.

38. \$57.46 and \$18.00 + \$24.25.

39. \$50.20 + \$4.01 and \$76.31.

40. \$98.76 and \$30.46 + \$43.04.

41. 2762 + 10341 and 45701 + 1200 ?

42. 3000 + 42301 and 720 + 1684 + 7342 ?

43. A merchant bought a quantity of goods for \$1248.65, and sold them for \$1540. What did he gain?

44. Sold a horse for \$250.75, which was \$28 more than he cost. What did he cost?

45. A man having \$15740.80, gave \$5085 for a store, and \$7640.75 for goods. How much money had he left ?

46. If a piece of property bought for \$7086.86 is sold at a loss of \$1562.09, for how much is it sold ?

Find the *second member* of the following equations:

47. $12346+840+1046-3846=?$

48. $\$210+\$809.76-(\$15.21+\$308.76)=?$

49. $\$600.09-\$276.25+(\$5682-\$654)=?$

50. $\$1032.07+\$68.05+\$.98-\$1000=?$

51. $476281-12672-8720+20000=?$

52. $30672+(84200-9264)-15000=?$

REVIEW.

ORAL EXERCISES.

151. *1.* The sum of two numbers is 46, and one of them is 18; what is the other?

2. The difference of two numbers is 16, and the greater is 32; what is the less?

3. A lady having 3 ten-dollar bills and 1 five-dollar bill, bought a bonnet for $11, a pair of gaiters for $7, and a scarf for $3. How much money had she left?

4. A man died at the age of 64 years, having been married 36 years. What was his age when he married?

5. In a public school there are 75 pupils, and 47 of them are girls; how many of them are boys?

6. A man sold 25 sheep, then bought 12, and then had 20. How many had he at first?

7. A merchant gave $52 for a box of goods, and paid $5 freight; for how much must he sell them to gain $15?

8. A man gave his watch and $10 in money for a harness valued at $75. How much did he get for his watch?

9. A man having received $45 for labor, paid $15 for a coat, $7 for a barrel of flour, and $6 for a ton of coal. How much had he left?

10. A man bought a vest for $7, a pair of pants for $12, and three shirts for $9, and gave in payment 3 ten-dollar bills. How much change should he receive?

Find the *required* term in the following equations:

11. $42-(10+12)=?$
12. $9+16=30-?$
13. $36-14=15+?$
14. $36-(8+9)+12=?$
15. $7+16-8=22-?$
16. $14+\overline{28-16}-9=?$

WRITTEN EXAMPLES.

152. *1.* The subtrahend is 260346, and the remainder 72304. What is the minuend?

2. The difference is $310.62, and the minuend $1206.28. What is the subtrahend?

3. What is the sum of 4062 and 12356 increased by the difference between 15000 and 975?

4. From the sum of 23462 and 9030, subtract the difference between 34000 and 7640.

5. From the difference between 19876 and 6032, subtract the difference between 12000 and 673.

6. From what sum must $.62 be taken to leave a remainder of $14.60?

7. There were 67374 miles of railway in the United States in 1872, and 71564 miles in 1873. What was the gain in one year?

8. A man has $10000. How much must he add to this, to be able to pay for a farm worth $13640?

9. California contains 188981 square miles, and Texas 274356 square miles. How much larger is Texas than California?

10. Mt. Blanc is 15784 feet high, and Pike's Peak 14147 feet. What is the difference in their height?

11. A man willed $125000 to his wife and two children. To his son he gave $44675, to his daughter $26380, and the remainder to his wife. What was his wife's share?

12. A merchant of Nashville goes to New Orleans with $21600. He invests $7638.50 in groceries, $3210.65 in crockery, $1245.18 in woodenware, and the remainder in hardware. How much does he invest in hardware?

13. What number increased by the difference between 7032 and 4100, will make 14062 + 458 ?

14. What is the result of 8472 + 602 + 72, diminished by 472 + 1020 + 125 ?

15. Mr. Jones having $12470, paid $4070 for a house, $927 for furniture, and $1000 for improvements. How much money had he left ?

16. A gentleman possessing $47840, at his death gave to each of his two sons $18755, and the remainder to his daughter. How much did the daughter receive ?

17. A grain dealer bought 5075 bushels of wheat at one time, and 12300 bushels at another. He sold 3763 bushels to one customer, and 4000 bushels to another. How many bushels had he left ?

18. The sum of four numbers is 847621; the first is 44000, the second is 12375 less than the first; the third is 2400 more than the second: what is the fourth ?

19. If a man's income is $10000 a year, and he pays $1500 for rent, $1250 for provisions, $375 for taxes, and $1463 for other expenses, how much will he have left ?

20. A man bought a western farm for $22700; he expended $4375 for improvements, and $2862 for stock: he sold the whole for $32000. Did he gain or lose, and how much ?

21. A man owning a farm of 325 acres and another of 430 acres, bought a half section and then sold 270 acres. How many acres had he left ?

22. From the sum of the days in the last six months of the year, subtract 175 days and add to the result 85 days. What is the final result ?

23. A man invests in trade at one time $680; at another time $820; at a third time $1580; and on a fourth occasion $420. How much must he add to the sum of these that the amount may be $5000?

24. A speculator gained $5760, and afterward lost $2746; at another time he gained $3575, and then lost $4632. How much did his gains exceed his losses?

25. A merchant commenced business with a capital of $12000. The first year he gained $2500; the second, he lost $1500, and the third, he gained $1865.75. What was his capital then, and how much had he gained?

26. The population of London in 1870 was 3250000; of New York, 944292; and of Brooklyn, 396099. How much greater was the population of London than of New York and Brooklyn?

27. A man owns property valued at $75860, of which $45640 is invested in real estate, $25175.75 in personal property, and the remainder he has in bank. How much has he in bank?

28. Three persons bought a hotel valued at $42075. The first agreed to pay $8375.50, the second agreed to pay twice as much, and the third the remainder. How much was the third to pay?

29. A had $725.40, B had $180.36 more than A, and C had as much as A and B together minus $214. How much had C?

30. A man buys goods as follows: groceries for $9.68; dry goods for $7.49; clothing, $19.95; nails for $2.58; and a plow for $9.75, and pays for them with a 50-dollar bill. How much change does he receive?

30. A man owes to one creditor $275.84; to another, $460.75; and to another, $600; and he pays $1274.50. How much does he still owe?

31. 376 + 1684 + 573 − 931 + 1000 = ?

32. $27.62 − ($30.50 − $14.00) − $7.62 = ?

33. 17300 + 6840 − 5800 + 1386 = 25300 − ?

34. (48036 − 7690) − (3600 + 1873) = 18321 + ?

Review *drill exercises* as directed on Pages 26 and 58.

The following *form* of combining Addition and Subtraction will afford a useful exercise:

35. From 2535 take (1389 + 275 + 67 + 88).

$$\begin{array}{r} 2535 \\ 1389 \\ 275 \\ 67 \\ 88 \\ \hline 716 \end{array}$$

Write the numbers as in addition, the minuend at the top. Then add and subtract; thus, 8, 15, 20, 29 and 6 are 35; write the 6 in the remainder, and carry 3 to the next column. 3, 11, 17, 24, 32 and 1 are 33; write the 1 in the remainder, and carry 3 to the next column. 3, 5, 8 and 7 are 15; write the 7 in the remainder, and carry 1 to the next column. 1, 2 and no remainder: 716 is the remainder. Prove the work by adding the remainder and the several parts of the subtrahend together, and the sum will equal the minuend.

In the same manner solve the following:

(*36.*)	(*37.*)	(*38.*)	(*39.*)	(*40.*)
3678	70532	43105	84576	10000
1507	27006	10764	12507	4732
783	13873	8096	10683	2068
45	2567	300	24562	1876
9	140	87	7300	67

41. From 43700 take (26020 + 5846 + 1600 + 759).

42. From 80009 take (30476 + 20040 + 7803 + 1567 + 305).

MULTIPLICATION.

153. ***Multiplication*** is the process of taking one of two numbers as many times as there are units in the other. Or,

It is a short method of *adding equal* numbers.

154. The ***Multiplicand*** is the number to be multiplied.

155. The ***Multiplier*** is the number by which to multiply. It shows how *many times* the multiplicand is to be taken.

156. The ***Product*** is the result obtained by the multiplication.

For definition of *sign,* etc., see Page 29.

$8\times 8+10=?$

$9\times 4-10=?$

$12\times 6+15=?$

$8\times 11-12=?$

$9\times 9+19=?$

$10\times 10-14=?$

$7\times 12+16=?$

$8\times 0-7=?$

$0\times 12\times 8=?$

$1\times 12+8=?$

$11\times 11-9=?$

$12\times 0+25=?$

$10\times 12-16=?$

$12\times 11-12=?$

$12\times 12+6=?$

Similar dictation exercises should be given by the teacher.

157. PRINCIPLES.—*1. The multiplier is always regarded as an abstract number.*

2. The multiplicand and product are like numbers, and may be either concrete or abstract.

In examples containing concrete numbers, the concrete number is the *true multiplicand,* but when it is the smaller, it is often, for convenience, used *abstractly* as the multiplier.

ORAL EXERCISES.

158. *1.* At 7 cents each, what is the cost of 5 pencils?

ANALYSIS.—Since 1 pencil costs 7 cents, 5 pencils will cost 5 times 7 cents, or 35 tents.

2. What is the cost of 4 tons of coal, at $8 a ton?

3. What is the cost of 5 hats, at $5 apiece?

4. At 9 cents each, what will 3 melons cost?

5. What will 5 yards of gimp cost, at 11 cents a yard?

6. 12 inches make a foot. How many inches in 4 feet?

7. At $4 a cord, what will 9 cords of wood cost?

8. At 10 cents a pound, what will 9 pounds of sugar cost? 10 pounds? 11 pounds? 12 pounds?

9. What cost 6 pairs of boots, at $8 a pair? At $9?

10. At 8 cents each, what cost 9 books? 10 books?

11. What cost 9 barrels of flour at $9 a barrel? At $10?

12. 7 days make a week. How many days in 7 weeks?

13. If a man earn $12 in 1 week, how much will he earn in 8 weeks? In 9 weeks?

14. How many bushels of grain can be put in 8 bins, each bin containing 12 bushels?

15. How many are 8 times $4, minus $7?

16. How many are 7 times 9 pounds, plus 10 pounds?

17. How many are 6 times 12 rods, less 20 rods?

18. James gave 5 cents each for 6 oranges. How much change should he receive for 50 cents?

19. How much more than $35 will 7 tons of coal cost, at $6 a ton?

For further Oral and Drill Exercises, see Pages 32 and 33.

WRITTEN EXERCISES.

159. When the multiplier consists of but one order of units.

1. How many are 4 times 73 ?

1st operation.

$$\begin{array}{r} 73 \\ 73 \\ 73 \\ \underline{73} \\ \text{Sum} \quad 292 \end{array}$$

Analysis.—To obtain the result by Addition.

First find the sum of four 3's, or 4 *times* 3 units, which is 12 units, equal to 1 ten and 2 units. Write the 2 units in the units' place, and reserve the 1 ten to be added to the sum of the tens.

Next the sum of four 7's, or 4 *times* 7 tens, is 28 tens, plus 1 ten reserved are 29 tens, or 2 hundreds and 9 tens, which write in the hundreds' and tens' place. Hence the *sum* is 292.

2d operation.

$$\begin{array}{lr} \text{Multiplicand,} & 73 \\ \text{Multiplier,} & \underline{4} \\ \text{Product,} & 292 \end{array}$$

Analysis.—In this operation, the multiplicand 73 is written but once; and as it is to be taken 4 times, write the multiplier 4 under it, and commence at the right to multiply. 4 times 3 units are 12 units, or 1 ten and 2 units. Write the 2 units in units' place and reserve the 1 ten to add to the product of the tens.

Next, 4 times 7 tens are 28 tens, plus 1 ten reserved are 29 tens, or 2 hundreds and 9 tens, which write in the hundreds' and tens' places. Hence the *product* is 292, equal to the *sum* in the first operation.

Solve by both methods,

2. 3 times 84.
3. 4 times 135.
4. 5 times 234.
5. 6 times 352.
6. 4 times $204.
7. 5 times $425.

8. Multiply 4621 by 4; by 5; by 6; by 7.
9. Multiply 3062 by 6; by 7; by 8; by 9.

What is the product

10. Of $5642 by 6 ? by 5 ? by 7 ? by 9 ?
11. Of 20372 feet by 7 ? by 9 ? by 5 ? by 6 ?

160. When *either* factor contains cents, the *product* is *cents,* and may be changed to dollars and cents by putting the point (.) *two* places from the *right,* and prefixing the sign ($). (**73.**)

	(1.)	*(2.)*	*(3.)*	*(4.)*
Multiply	$43.72	$136.04	87 cents.	$2.06
By	8	7	9	6
Product	$349.76	$952.28	$7.83	$12.36

5. What will be the cost of 231 tons of coal at $9 a ton?

Although $9 is the *true multiplicand,* for convenience we may use it as the multiplier, and 231 as the multiplicand; but the *product* is *dollars* because the true multiplicand is dollars.

6. At 6 cents a pound, what cost 675 pounds of rice?

7. At $4.37 a yard, what is the cost of 7 yards of cloth?

8. At $124.50 an acre, what will 5 acres of land cost?

9. What is the cost of 8 building lots, at $2015 each?

10. What cost 326 tons of coal, at $6 a ton?

11. What cost 1238 cords of wood, at $5 a cord?

12. What cost 752 pounds of nails, at 7 cents a pound?

13. What cost 2140 barrels of flour, at $8 a barrel?

14. What cost 379 pounds of nails, at 6 cents a pound?

15. At 9 cents a pound, what will 1872 pounds of sugar cost?

16. At $98 a month, how much can a man earn in 8 months?

17. At $6.75, what will 7 tons of coal cost?

18. What are 514 barrels of apples worth at $4 a barrel? At $5? At $7?

19. What cost 3250 pounds of pork, at 6 cents a pound? At 8 cents? At 9 cents?

161. When the multiplier consists of two or more orders of units.

1. Multiply 78 by 46.

OPERATION.

Multiplicand,	78	
Multiplier,	46	
1st Partial Prod.	468	= 78 × 6
2d Partial Prod.	312	= 78 × 40
Entire Prod.	3588	= 78 × 46

ANALYSIS.—Write the numbers as before.

Since 46 is composed of 6 units and 4 tens, 46 times any number is equal to 6 times the number, *plus* 4 tens, or 40 times the number.

6 times 78 is 468, the first *partial* product. 4 tens times 8 units are 32 *tens*, or 3 hundreds and 2 tens. Write the 2 tens in the tens' place, in the second *partial* product, and reserve the 3 hundreds to add to the product of hundreds.

4 tens times 7 tens are 28 *hundreds*, plus 3 hundreds reserved, are 31 hundreds, or 3 thousands and 1 hundred, which write in the second *partial* product. The sum of the *partial* products is the *entire product* 3588.

RULE.—I. *Write the multiplier under the multiplicand, so that units of the same order stand in the same column.*

When the multiplier consists of one figure.

II. *Begin at the right and multiply the units of each order of the multiplicand by the multiplier. Write in the product the units of each result, and reserve the tens to add to the next result.*

When the multiplier consists of more than one figure.

III. *Multiply the multiplicand by the units of each order of the multiplier successively, beginning at the right, and write the right-hand figure of each partial product under the order of the multiplier used.*

The sum of the partial products is the required product.

PROOF.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results are the same, the work is probably correct.*

When there are ciphers in the multiplier, multiply by the significant figures only, since the product of any number by 0 is 0.

2. Multiply 6432 by 75; by 67; by 136.

3. Multiply 4372 by 25; by 48; by 67; by 128.

4. Multiply 3065 by 72; by 84; by 93; by 204.

What is the value

5. Of 67 hogsheads of sugar, at $7.75 a hogshead?

6. Of 2347 acres of land, at $136 an acre?

7. What will be the cost of building a line of telegraph 274 miles long, at $967 a mile?

8. If 1049 pounds of seed cotton are raised from an acre of land, how many pounds will 386 acres produce?

9. If a cotton mill manufactures 628 yards of cloth in a day, how many yards can it make in 297 days?

10. How many yards of shirting in 49 bales, each bale containing 26 pieces, and each piece 57 yards?

11. What is the cost of 128 barrels of beef, each containing 216 pounds, worth 13 cents a pound?

12. Three schooners, each, ship 239 cords of wood, and a fourth ships 248 cords. What is the value of the whole at $4.25 a cord?

13. If it require 108 tons of iron rail for 1 mile of track, how many tons will be required for 476 miles, and what will be its value at $145 a ton?

14. A crop of cotton was put up in 472 bales, the average weight of which was 588 pounds. What was the weight of the whole crop, and its value at 18 cents a pound?

162. To multiply by the factors of a number.

A ***Factor*** of a number is one of its *equal* parts.

Thus, 16 is composed of eight 2's, four 4's, or two 8's; hence 2, 4, and 8 are *factors* of 16.

The multiplicand and multiplier are called the *factors* of the product.

Thus, 12 × 9 = 108. The product 108 is composed of nine 12's or twelve 9's; hence 12 and 9 are *equal* parts, or *factors* of 108.

163. PRINCIPLE.—*The product of any number of factors will be the same in whatever order they are multiplied.*

1. Multiply 468 by 36.

OPERATION.

36 = 6 × 6, or 9 × 4, or 12 × 3.

468	468	468	468
36	6	9	12
2808	2808	4212	5616
1404	6	4	3
16848	16848	16848	16848

RULE.—I. *Separate the multiplier into two or more factors.*

II. *Multiply the multiplicand by one of the factors, the resulting product by another factor, and so continue until all the factors have been used.*

The last product will be the product required.

In like manner, multiply

2. $73.04 by 48 = 8 × 6.
3. 50076 by 72 = 6 × 4 × 3.
4. $206.14 by 96 = 4 × 4 × 6.
5. $780.91 by 108.
6. 140086 by 120.
7. $457.52 by 240.

8. What will 56 acres of land cost, at $164.50 an acre?

164. When either the multiplicand or multiplier, or both, have ciphers on the right.

1. Multiply 286 by 100.

OPERATION.

```
  286
  100
-----
28600
```

ANALYSIS.—Since removing a figure *one place* to the *left*, increases its value *ten times* (**126**), annexing a cipher to a number multiplies it by 10; annexing *two* ciphers multiplies it by 100, etc. Hence 286 × 100 = 28600, the product required.

2. Multiply 3240 by 600.

OPERATION.

```
 3240
  600
-------
1944000
```

ANALYSIS.—3240 = 324 × 10, and 600 = 6 × 100. First multiply together the two factors 324 and 6, and then multiply their product 1944 by 10 × 100, or by 1000, by annexing *three* ciphers, which gives 1944000, the required product.

What is the product

3. Of 372 by 10? By 100? By 1000? By 10000?

4. Of 860 by 50? By 400? By 1500? By 3000?

RULE.—*To the product of the significant figures, annex as many ciphers as there are ciphers on the right of both of the factors.*

What is the product

5. Of $4.72 by 100?

6. Of $30.40 by 60?

7. Of $1200 by 700?

Find

8. 120 times 5000.

9. 600 times 21000.

10. 1000 times 104000.

11. 42030090 × 3020 = ?

12. 7000600 × 50040 = ?

13. There are 640 acres in 1 square mile. How many acres in 150 square miles? In 200? In 420?

14. The salary of the President is $50000 a year. How much does he receive in 8 years?

15. If there be 530 shingles in a bunch, how many shingles in 2700 bunches?

REVIEW.

ORAL EXERCISES.

165. *1.* The sum of $8+12+16$ equals the product of $9 \times$ what number?

2. The sum of $40 - 14$ and $12 + 4$ equals $7 \times$ what number?

3. The difference between $35+15$ and $24-10$ is equal to the product of what two factors? Three factors?

4. The product of what two factors is equal to the sum of 9, 20, and 11?

5. A woman sold a grocer 4 dozen of eggs at 24 cents a dozen, and received in payment half a pound of tea worth 50 cents, and 2 pounds of sugar at 11 cents a pound. How much was still due her?

6. A boy bought a book for 36 cents, a slate for 20 cents, and a pencil for 4 cents. How much change should he receive for a 1 dollar bill?

7. A lady bought 9 yards of silk at $3 a yard, 3 pairs of kid gloves at $2 a pair, 4 pairs of hose at half a dollar a pair. She gave in payment 4 ten-dollar bills. How much change should she receive?

Find the *required term* in the following equations:

8. $4 \times 8 + 7 - 9 = ?$
9. $5 \times 3 \times 2 + 8 = ?$
10. $10 - 3 + (5 \times 6) = ?$
11. $\overline{7 \times 6} + \overline{3 \times 6} = ?$
12. $(4 \times 10) - (4 \times 5) = ?$
13. $8 \times 5 + 6 + 10 = ?$

14. $\overline{8 \times 4} - \overline{6 \times 0} = ?$
15. $4 \times 10 + 3 + ? = 50$
16. $0 \times 7 + 8 + ? = 40$
17. $12 + 4 - 8 \times ? = 32$
18. $9 \times 8 + 10 - ? = 75$
19. $48 - 20 + (4 \times ?) = 40$

166. By a little practice numbers containing *three* or *four* figures may be multiplied *mentally*, by first multiplying the highest order of units, and adding the product of each lower order as found.

1. Multiply 324 by 2.

OPERATION.—2 times 3 hundreds are 600; 2 times 2 tens are 4 tens, or 40, and 600 + 40 are 640; 2 times 4 are 8, and 640 + 8 are 648.

Omitting all but results, the required product will be easily and promptly obtained by a strictly mental process. Thus, 600, 640, 648.

In like manner, find the product of

2. 3 times 230.
3. 3 times 342.
4. 4 times 150.
5. 6 times 241.
6. 234 × 2.
7. 501 × 3.

WRITTEN EXERCISES.

167. *1.* If I receive $1500 salary, and pay $370 for board, $281.50 for clothing, $112.75 for books, and $196.65 for other expenses annually, what can I save in 3 years?

2. A merchant bought 7 hogsheads of sugar at $46.45 a hogshead, and sold it for $53.62 a hogshead. What did he gain?

3. Paid $2709 for 388 barrels of flour, and sold the same at $9.12 a barrel. What was the gain?

4. If a man have an income of $5670 a year and his daily expenses average $7.25, how much can he save in a year of 365 days?

5. What sum must be added to 272 × 400, to make the amount 126720?

6. A man owing $15760, gave in payment 5 lots of land, worth $730 each, 5 horses, valued at $236.50 each, an interest he had in a coal mine worth $2000, and $1728.75 in money. How much remained unpaid?

7. A flour merchant bought 240 barrels of flour for $1920, and sold the same at $10 a barrel. What did he gain?

8. Bought 36 tubs of butter, each weighing 108 pounds; the tubs alone weighing 21 pounds each. What is the weight of the butter without the tubs?

9. From 227 × 87 × 15 subtract 406 × 327.

10. What sum must be added to 356 × 264 to make the amount 3447 less than 243042?

11. What is the difference between 26460 — (487 × 37) and 105 × 246 — 12075?

12. From 207300 — (1236 × 48) take 976 × 98 + 10050.

13. A man bought 45 acres of land at $38 an acre, and 76 acres at $47 an acre, and sold the whole at $45 an acre. Did he gain or lose, and how much?

14. A farmer exchanged 584 bushels of wheat at $2 a bushel, for 78 barrels of flour at $9 a barrel, and received the balance in money. How much money did he receive?

15. Two persons start from the same point and travel in opposite directions; one travels at the rate of 32 miles a day, the other at the rate of 39 miles a day. How far apart will they be in 14 days?

16. A man bought two farms; one of 157 acres at $26 an acre, and another of 138 acres at $34 an acre. He paid at one time $4725, and at another $1895; how much remained unpaid?

17. A planter sold 209 bales cotton at $76 a bale, and from the proceeds he bought 107 acres of land at $60 an acre, 18 mules at $75 each, and 4 span of horses at $218 a span. How much money had he left?

18. If a man buy 40 acres of land at $35 an acre, and 56 acres at $29 an acre, and sell the whole for $32 an acre, what does he gain or lose?

19. Bought 360 barrels of flour for $2340, and sold the same at $8 a barrel; what was gained by the bargain?

20. A farmer sold 462 bushels of wheat at $2 a bushel, for which he received 75 barrels of flour at $9 a barrel, and the balance in money; how much money did he receive?

21. A man bought two farms, one of 136 acres at $28 an acre, and another of 140 acres at $33 an acre; he paid at one time $4000, and at another time $1875; how much remained unpaid?

22. In an orchard, 76 apple trees yield 18 bushels of apples each, and 27 others yield 21 bushels each; what are the apples worth at 30 cents a bushel?

23. A farm-house is worth $3246, the farm is worth 3 times as much plus $1200, and the stock is worth twice as much as the house, less $1875. What is the value of the whole, and of the farm and stock?

24. What is the difference in the cost of 48 horses at $184.50 each, and 130 sheep at $4.80 a head?

25. Bought 150 barrels of flour for $1150, and finding 25 barrels of it worthless, sold the remainder at $9 a barrel. Did I gain or lose, and how much?

26. $(142 + 405) \times (1000 - 850) - 5000 = ?$

27. $(97 \times 1000) - (75 \times \overline{500 - 420}) + 1500 = ?$

28. $\$73.46 - (\$.94 + \$3.02) + (\$47 \times 35) = ?$

29. $(\$246.08 \times 104) + (\$2000 - \$240.50) \times 10 = ?$

DRILL EXERCISES.

168. *1.* 1 × 1, 2 × 2, 3 × 3, etc., to 25 × 25.

2. 1 × 1 × 1, 2 × 2 × 2, 3 × 3 × 3, etc., to 12 × 12 × 12.

3. 10 × 10, 20 × 20, 30 × 30, etc., to 90 × 90.

4. 10 × 10 × 10, 20 × 20 × 20, etc., to 90 × 90 × 90.

5. 1 × 100, 2 × 200, 3 × 300, etc., to 9 × 900.

6. 10 × 100, 20 × 200, 30 × 300, etc., to 90 × 900.

7. 100 × 100, 200 × 200, 300 × 300, etc., to 900 × 900.

8. Multiply 20, 200, 2000, each by all the numbers from 3 to 9 inclusive.

9. Multiply 30, 300, 3000, by same multipliers as in example 8.

10. In the same manner proceed with all the similar numbers to 90, 900, 9000.

11. In columns A, B, C, D, and E, page 94, multiply rapidly by 10, 20, 30, etc., to 90.

12. In the same columns multiply rapidly by 100, 200, etc., to 900.

The above will furnish abundant exercise in rapid oral work.

These combinations should be made from dictation as well as at sight. Require multiplication at sight, as on Page 32.

For slate work dictate examples as follows:

13. Find product of 17, 19, and 21. Of 20, 30, and 27.

14. Of all the numbers from 21 to 26.

Familiarity with the oral exercises above given will give the pupil such facility in the combination of numbers as to make his future progress easy and rapid.

Insist upon promptness, rapidity, and accuracy. Do not continue the work until it drags.

Review Drill Table, on page 94.

DIVISION.

169. ***Division*** is the process of finding *how many times* one number is contained in another of the same kind, or of finding *one* of the *equal parts* of a number.

170. The ***Dividend*** is the number to be divided.

171. The ***Divisor*** is the number by which to divide.

172. The ***Quotient*** is the result of the division, and shows *how many times* the dividend contains the divisor.

For definition of *remainder, sign* of division, etc., see Page 38.

36 ÷ 9 = ?   63 ÷ 9 = ?   64 ÷ 8 = ?   84 ÷ 7 = ?
42 ÷ 7 = ?   56 ÷ 8 = ?   66 ÷ 6 = ?   72 ÷ 12 = ?
40 ÷ 5 = ?   45 ÷ 5 = ?   72 ÷ 9 = ?   96 ÷ 8 = ?

173. PRINCIPLES.—In finding *how many times* one number is contained in another:

1. *The divisor and dividend are like numbers, and the quotient an abstract number.*

In finding one of the *equal parts* of a number:

2. *The dividend and quotient are like numbers, and the divisor an abstract number.*

3. *The dividend is equal to the product of the divisor by the quotient, plus the remainder.*

174. Division may also be regarded as a short method of performing several *subtractions* of a number.

Thus, $24 - 6 = 18$; $18 - 6 = 12$; $12 - 6 = 6$; $6 - 6 = 0$. We have performed *four* subtractions of 6, hence there are *four* 6's in 24, or 6 is contained in 24, 4 times.

175. Since one number is contained in another as many times as it is a *factor* of the other, division may be regarded as the *reverse* of multiplication.

176. In Multiplication, *both factors* are given to find the *product;* in Division, *one factor* and the *product* (answering to the *dividend*) are given to find the *other factor,* which answers to the *quotient.*

Thus, $6 \times 4 = 24$, the factor 6 being taken 4 times; hence there are *four* 6's in 24, or 6 is contained in 24, 4 times.

177. The *equal parts* into which a unit or whole thing is divided are called *fractions.*

The *names* of the equal parts of a unit vary according to the *number* of the parts; thus, *one-half* is one of *two* equal parts, *one-third* is one of *three* equal parts, into which the whole thing or number is divided.

So in like manner we have *fourths, fifths, sixths, sevenths, eighths, tenths, twelfths, twentieths,* etc.

178. These parts are expressed by writing the number denoting the *name* of the parts below a short horizontal line as a *divisor,* and the *number* of parts taken or used, above the line as a *dividend.*

Thus, $\frac{1}{2}$ signifies 1 *divided by* 2, and is read, *one-half.*

$\frac{2}{3}$ signifies 2 *divided by* 3, and is read, *two-thirds.*

$\frac{7}{12}$ signifies 7 *divided by* 12, and is read, *seven-twelfths.* etc.

179. Find the quotients of the following. **(Art. 38.)**

$\frac{63}{7} = ?$ $\frac{96}{8} = ?$ $\frac{77}{7} = ?$ $\frac{120}{10} = ?$
$\frac{72}{9} = ?$ $\frac{70}{7} = ?$ $\frac{96}{12} = ?$ $\frac{108}{12} = ?$
$\frac{48}{12} = ?$ $\frac{84}{12} = ?$ $\frac{72}{6} = ?$ $\frac{132}{11} = ?$

1. Find one-half of 2, one-half of 4, one-half of 6, one-half of 8, and so on to one-half of 30.

2. Find one-third of 3, one-third of 6, one-third of 9, one-third of 12, and so on to one-third of 30.

3. Find $\frac{1}{4}$ of 4, $\frac{1}{4}$ of 8, $\frac{1}{4}$ of 12, $\frac{1}{4}$ of 16, to $\frac{1}{4}$ of 40.

4. Find $\frac{1}{5}$ of 5, $\frac{1}{5}$ of 10, $\frac{1}{5}$ of 15, $\frac{1}{5}$ of 20, to $\frac{1}{5}$ of 50.

5. How do you find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc., of any number?

6. How many yards of cloth, at $4 a yard, can be bought for $36?

ANALYSIS.—As many yards as $4 is contained times in $36, which is 9 times. Hence 9 yards can be bought for $36.

7. At $6 a ton, how many tons of coal can be bought for $24? For $30? For $54? For $72?

8. If 7 cords of wood cost $42, what does 1 cord cost?

ANALYSIS.—Since 7 cords of wood cost $42, 1 cord costs 1 *seventh* of $42, or $6. Hence 1 cord costs $6.

9. If a man travel 48 miles in 4 hours, how far does he travel in 1 hour?

10. Find the cost of 1 ton of coal, if 8 tons cost $64?

11. A man sold 8 bushels of cranberries for $32. How much did he receive a bushel for them?

12. A farmer gathered 108 bushels of apples from 9 trees. What was the number of bushels to each tree?

13. A merchant paid $96 for 8 pieces of dress goods. What was the cost of each piece?

For further Oral and Drill Exercises see Pages 52 and 53.

WRITTEN EXERCISES.

180. When the divisor consists of but one order of units.

1. Divide 875 by 7.

OPERATION.

$7\,)\underline{\,8\,7\,5\,}$

Quotient $\quad 1\,2\,5$

ANALYSIS.—7 is contained in 8, once, and 1 remainder. 1 prefixed to 7 makes 17. 7 is contained in 17, 2 times and 3 remainder. 3 prefixed to 5 makes 35, and 7 is contained in 35, 5 times. Hence the quotient is 125.

In like manner, divide and analyze the following:

(*2.*)	(*3.*)	(*4.*)	(*5.*)
$6\,)\underline{\,7944\,}$	$7\,)\underline{\,9464\,}$	$8\,)\underline{\,8928\,}$	$5\,)\underline{\,6895\,}$

6. Divide 92352 by 8; by 6; by 4.

7. Divide 83762 by 7; 79880 by 6; 3263 by 8.

OPERATION.

$8\,)\underline{\,3\,2\,6\,3\,}$

Quotient $\quad 4\,0\,7\,\frac{7}{8}$

ANALYSIS.—Since 8 is not contained in 3 thousands, unite the 3 thousands and 2 hundreds, making 32 hundred. 8 is contained in 32 hundred, 4 hundred times, which write in the hundreds' place in the quotient. Next, 8 is not contained in 6 tens, so write a cipher in tens' place in the quotient, and unite the 6 tens and 3 units. 8 is contained in 63 units 7 times and 7 units remainder, which write over the divisor and add as a part of the quotient. Hence the quotient is $407\frac{7}{8}$.

PROOF.—Multiply the quotient 407 by the divisor 8, and the product is 3256; 3256 plus the remainder 7, equals the dividend 3263. (PRIN. 3.)

8. Divide 8135464 by 6; by 8; by 7; by 5; by 9.

9. Divide $48.56 by 8 cents.

OPERATION.

$\$.08\,)\underline{\,\$48.56\,}$

$\quad 6\,0\,7$ times.

Eight cents may be written $.08 (71). When the divisor and dividend are like numbers, the quotient is an abstract number (PRIN. 1). Hence 8 cents are contained in $48.56, 607 *times*.

10. Divide $48.56 by 8.

OPERATION.
8) $48.56
$6.07

When the divisor is an abstract number, the dividend and quotient are like numbers (PRIN. 2). Hence 1 *eighth* of $48.56 is $6.07.

PROOF.—$6.07 × 8 = $48.56.

Solve and prove,

(*11.*)	(*12.*)	(*13.*)	(*14.*)
9) $217.62	7) $6.44	$7) $644	$.07) $6.44
$24.18	$.92	92 times.	92 times.

How many times

15. Are $8 contained in $15096? In $58424? In $23064?

16. Is 7 contained in 330457? In 19728? In 918271?

17. Is 9 contained in 436281? In 605675? In 1039126?

Find

18. 1 *fifth* of $863.25.

19. 1 *sixth* of 34807 tons.

20. 1 *eighth* of 20673 days.

21. 1 *ninth* of $7384.50.

What is

22. $\frac{1}{7}$ of 500322 miles?

23. $\frac{1}{4}$ of 32876 men?

24. $\frac{1}{3}$ of 60349 acres?

25. $\frac{1}{12}$ of 760344 rods?

26. How many barrels of flour at $8 a barrel, can be bought for $12736? For $7068?

27. If 75000 bushels of grain are put into 8 bins of equal size, how many bushels does each bin contain?

28. If 9 acres of land cost $976.50, what is the cost of 1 acre?

29. How many oranges can be bought for $3.72, at 4 cents apiece?

30. At 8 cents a yard, how many yards of ribbon can be bought for $7.28?

31. Paid $1792 for 7 horses. What did each cost?

181. When the divisor consists of more than one order of units.

1. Divide 5437 by 26.

OPERATION.
Divisor. Dividend. Quotient.
26)5437(209$\frac{3}{26}$
52
237
234
3 Remainder.

ANALYSIS.—26 is contained in 54 hundred, 2 hundred times, with a remainder. Write the 2 hundreds in the quotient, and multiply the divisor 26 by this quotient figure, and subtract the product, 52 hundreds, from 54 hundreds, the *first* partial dividend, and there remains 2 hundreds. To this annex the 3 tens of the dividend, making 23 tens for the *second* partial dividend.

26 is not contained in 23, so write a cipher in the quotient and bring down the 7 units of the dividend, making 237 units for the *third* partial dividend.

26 is contained in 237 units 9 times, with a remainder. Write the 9 units in the quotient, and multiplying and subtracting as before, there remain 3 units, which write over the divisor, and annex as a part of the quotient. Hence the quotient is 209$\frac{3}{26}$.

PROOF.—$209 \times 26 + 3 = 5437$.

182. ***Long Division*** is the process of dividing when the subtractions *are written*, and consists of five principal steps, viz.:

1st. Writing down the numbers. 3d. Multiplying.
2d. Finding how many times. 4th. Subtracting.
5th. Bringing down another figure.

How many times

2. Is 13 contained in 169 ? In 273 ? In 4550 ?

3. Is 14 contained in 196 ? In 1596 ? In 3304 ?

4. Is 15 contained in 2630 ? In 7645 ? In 18306 ?

5. Is 17 contained in 238 ? In 2465 ? In 50571 ?

6. Is 18 contained in 7362 ? In 10404 ? In 11052 ?

7. At $17 a barrel, how many barrels of molasses can be bought for $8024 ?

8. Divide 32762 by 14; by 16; by 23; by 28.

9. Divide 130426 by 58; by 63; by 81; by 74.

RULE—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the least number of the left hand orders of the dividend that will contain it, and write the result for the first figure of the quotient.*

III. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the figure of the next lower order of the dividend for a new partial dividend, and divide as before.*

IV. *Proceed in the same manner until all the orders of the dividend have been used.*

V. *If any partial dividend does not contain the divisor, write a cipher in the quotient, and annex the next order of the dividend, and proceed as before.*

VI. *If there be at last a remainder, write it after the quotient with the divisor, underneath.*

PROOF. *Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will be equal to the dividend.*

1. If the product of the divisor and quotient be *greater* than the *partial* dividend, the quotient is *too large*, and must be *diminished*.

2. If any remainder is *equal to* or *greater* than the divisor, the quotient is *too small*, and must be *increased*.

183. When the divisor and dividend are both concrete numbers, they must be of the *same name*. Hence, if one is *dollars*, and the other *cents* or dollars and cents, before dividing, change so that both may be *cents*.

184. Since 100 cents make 1 dollar, there are 100 times as many cents as dollars. Hence,

To change a number representing dollars to a number representing cents, annex two ciphers, omit the sign ($), and write the word *cents* after it.

To change dollars and cents to the same form, omit the sign ($) and the point (.), and write the word *cents* at the right.

1. Divide $46.92 by 23.

OPERATION.

$$\begin{array}{l} 23)\$46.92(\$2.04 \\ \quad\;\; \underline{46} \\ \quad\quad\quad 92 \\ \quad\quad\quad \underline{92} \end{array}$$

2. Divide $46.92 by 23 cents.

OPERATION.

$$\begin{array}{l} 23)4692(204 \text{ times.} \\ \quad\; \underline{46} \\ \quad\quad 92 \\ \quad\quad \underline{92} \end{array}$$

3. Divide $46.92 by $23.

OPERATION.

$$\begin{array}{l} 2300)4692(2\tfrac{92}{2300} \text{ times.} \\ \quad\quad \underline{4600} \\ \quad\quad\quad 92 \end{array}$$

4. Divide $46 by 23 cents.

OPERATION.

$$\begin{array}{l} 23)4600(200 \text{ times.} \\ \quad\; \underline{46} \\ \quad\quad 00 \end{array}$$

In like manner divide, and prove the following:

5. $325.72 by 34.
6. $938.07 by 63.
7. $3176.46 by 126.
8. $49.56 by 14 cents.
9. $87.36 by 21 cents.
10. $288.96 by $.43.
11. $810.98 by $.46.
12. $594 by 18 cents.
13. $1385 by $105.
14. $32.48 by $7.

How many times

15. Is 47 contained in 30176 ? In 27865 ? In 103474 ?

16. Is 185 contained in 200376 ? In 4701625 ?

17. The annual receipts of a company are $570685. What is the average a day, if there are 313 working days ?

185. To divide by the factors of a number.

1. Divide 644 by 28, using the factors.

OPERATION.

4)644
7)161
23

ANALYSIS.—Since 28 is equal to 4 times 7, either divide by 28, or by its factors 4 and 7. Now, 644 ÷ 4 = 161; but this quotient is 7 times too great, and must therefore be divided by 7; hence, 161 ÷ 7 = 23, the true quotient.

		Factors.
2. Divide	6228 by	36, or by 4, and 9.
3. Divide	27360 by	96, or by 3, 4, and 8.
4. Divide	526050 by	126, or by 2, 7, and 9.
5. Divide	73416 by	168, or by 4, 6, and 7.

6. Divide 5831 by 84, using the factors 3, 4, and 7.

OPERATION.

3)5831
4)1943 2
7)485 . . 3 × 3 = 9
69 . 2 × 4 × 3 = 24
True Remainder. 35
69$\frac{35}{84}$ Quotient.

ANALYSIS. — Since 84 is equal to 3 × 4 × 7, divide by 84, or by its factors 3, 4, and 7.

5831 ÷ 3 = 1943, and a remainder of 2, which being a part of the dividend, is also a part of the true remainder.

1943 ÷ 4 = 485, and a remainder of 3. Since a unit of the first quotient 1943, equals 3 units of the dividend, this second remainder 3 being a part of 1943, equals 3 × 3, or 9 units of the dividend.

485 ÷ 7 = 69, and a remainder of 2. Since a unit of the second quotient 485 equals 4 units of the first quotient 1943, this third remainder 2 being a part of 485, equals 2 × 4 × 3, or 24 units of the dividend. Hence the first *partial* remainder is 2, the second is 9, the third is 24, and the *true* remainder 35; and the quotient 69$\frac{35}{84}$.

7. Divide 139074 by 72, using its factors, 3, 4, and 6.

8. Divide 7360479 by 96, using its factors, 2, 6, and 8.

Rule.—I. *Separate the divisor into two or more factors.*

II. *Divide the dividend by one of these factors, and the quotient thus obtained by another factor, and so on until all the factors have been used as divisors.*

III. *If there be remainders, multiply each remainder by all the divisors preceding the one that produced it.*

IV. *Add the products and the remainder from the first division, if any, and the sum will be the true remainder.*

9. Divide 376875 by 315, or by 5, 7, and 9.
10. Divide 73522 by 135, or by 3, 5, and 9.
11. Divide 401976 by 245, or by 5, 7, and 7.

186. When the divisor has ciphers on the right.

1. Divide 4067 by 10.

OPERATION.
1|0)406|7
406 . . 7 Rem.
$406\frac{7}{10}$ Quotient.

Analysis.—Since removing any order of figures *one place* to the *right*, diminishes its value *ten times* (**127**), by cutting off, or taking away, the right-hand figure of a number, each of the remaining figures, being removed one place to the right, is diminished in value ten times, or *divided by* 10.

For similar reasons, cutting off *two* figures divides by 100, cutting off *three* figures divides by 1000, and so on. The remaining figures are the quotient, and those cut off, the remainder.

Divide

2. 37684 by 100.
3. 103076 by 1000.
4. 267104 by 10000.
5. 5023001 by 100000.

6. How many watches at $100 each, can be bought for $6500 ?

7. Bought 1000 barrels of flour for $9750. What was the cost a barrel?

8. Divide 2416700 by 6000.

OPERATION.

6|000)2416|700

402 . . 4700 Rem.

$402\frac{4700}{6000}$ Quotient.

ANALYSIS. — Resolve 6000 into the factors 1000, and 6. First divide by 1000, by cutting off the three right-hand figures of the dividend. The quotient is 2416, and a remainder of 700. Next divide 2416 by 6; the quotient is 402 and a second remainder of 4 thousands, which prefixed to the *first* remainder 700 gives a *true* remainder of 4700. Hence the quotient is $402\frac{4700}{6000}$.

RULE.—I. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend.*

II. *Divide the remaining part of the dividend by the remaining part of the divisor.*

III.—*Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

9. If it require $34400 to pay a regiment of 800 men, how much does each man receive?

10. At $3400, how many lots can be bought for $68000?

11. How many bales, each weighing 470 pounds, can be made of 39500 pounds of cotton?

GENERAL PRINCIPLES OF DIVISION.

187. The quotient depends upon the *relative* values of the dividend and divisor. Hence, any change in the value of either dividend or divisor, will produce a change in the value of the quotient. But some changes may be made upon both dividend and divisor, which will not affect their *relative* values, and consequently will not affect the quotient. To illustrate, let $54 \div 9 = 6$, be the fundamental equation, with which the following are to be compared:

1. $(54\times3)\div9=162\div9=18$.	**Multiplying the dividend by 3 multiplies the quotient by 3.**
2. $54\div(9\div3)=54\div3=18$.	**Dividing the divisor by 3 multiplies the quotient by 3**
3. $(54\div3)\div9=18\div9=2$.	**Dividing the dividend by 3 divides the quotient by 3.**
4. $54\div(9\times3)=54\div27=2$.	**Multiplying the divisor by 3 divides the quotient by 3.**
5. $(54\times3)\div(9\times3)=162\div27=6$.	**Multiplying both dividend and divisor by 3 does not change the quotient.**
6. $(54\div3)\div(9\div3)=18\div3=6$.	**Dividing both dividend and divisor by 3 does not change the quotient.**

These six equations illustrate the following

188. General Principles of Division.

1. *Multiplying the dividend,* or *Dividing the divisor,*	*Multiplies the quotient.*
2. *Dividing the dividend,* or *Multiplying the divisor,*	*Divides the quotient.*
3. *Multiplying or dividing both dividend and divisor by the same number,*	*Does not change the quotient.*

These *three principles* may be embraced in one

GENERAL LAW.

189. *A change in the dividend produces a* LIKE *change in the quotient, but a change in the divisor produces an* OPPOSITE *change in the quotient.*

GENERAL REVIEW.

ORAL EXERCISES.

190. *1.* The sum of two numbers is 25, and one of the numbers is 9: what is the other?

2. The difference of two numbers is 12, and the smaller is 10: what is the larger?

3. The difference of two numbers is 9, and the larger is 30: what is the smaller?

4. The product of two numbers is 108, and one of the numbers is 12: what is the other?

5. The quotient of two numbers is 30 and the divisor 12: what is the dividend?

6. The dividend is 120 and the quotient 12: what is the divisor?

7. The quotient is 12, the remainder 6, and the divisor 10: what is the dividend?

8. A boy bought 12 oranges for 42 cents, and sold them so as to gain 18 cents. For how much a piece did he sell them?

9. How many pounds of rice at 6 cents a pound, will cost as much as 8 pounds of meat at 9 cents a pound?

10. If a man earn $5 while a boy earns $2, how many dollars will the man earn while the boy is earning $18.

11. How far will a man travel in 12 days at the rate of 36 miles in 3 days.

12. John gave 40 cents for some peaches, at the rate of 5 for 10 cents. How many peaches did he buy?

13. What is the value of 5 tons of hay, at the rate of $48 for 4 tons?

14. A tailor bought 11 yards of one kind of cloth for \$55, and 9 yards of another kind for \$36. What was the difference in the price per yard?

15. Multiply 10 by 8, subtract 30, add 13, divide by 7, multiply by 6, and what is the result?

16. What cost 28 quarts of milk, at 40 cents for 5 quarts?

17. Divide 60 by 12, multiply the quotient by 9, to the product add 25, from the sum subtract 40, and what will remain?

18. How many pairs of boots at \$8 a pair, will pay for 6 weeks board at \$7 a week, and \$22 borrowed money?

19. From 25 subtract 10, add 17, divide by 8, multiply by 20, subtract 14, divide by 6, multiply by 4, add 18, and what will be the sum?

20. Multiply 15 by 5, add 5, divide by 8, multiply by 6, subtract 10, divide by 5, add 2, multiply by 11, and what will be the product?

21. $10 + 8, \div 6, \times 9, - 7, \times 5, + 20, \div 10, + 8 = ?$

22. $25, - 5, \div 2, \times 9, + 10, \div 20, \times 12, + 15, - 5, \div 7 = ?$

23. $8 \times 9, \div 12, + 3, \times 5, + 10, \div 11, \times 8, + 20, \div 12, + 4, \times 7, - 3 = ?$

Find the missing *term* in each of the following equations:

24. $10 + 4 \div 7 + 20 = ?$
25. $8 \times 9 - 12 \div 5 = ?$
26. $\overline{40 - 8 \div 4} \times 7 = ?$
27. $(25 + 10) \div 7 + 9 = ?$
28. $72 \div 12 + 50 - 16 = ?$
29. $48 + 12 - 10 \div ? = 5.$
30. $108 \div 9 - 4 \times ? = 72.$
31. $14 + 7) \div 3 \times ? = 84.$
32. $(144 \div 12 - 3) \times 11 = ?$
33. $24 + 20 - 4 = 80 \div ?$
34. $\overline{56 \div 7} \times 8 = 70 - ?$
35. $43 + 6) \div 7 \times 0 = ?$
36. $120 \div 10 \times 5 = ? \times 12.$
37. $20 \times 2 \div 8 = ? \div 6.$

WRITTEN EXAMPLES.

191. *1.* How many times is 246 contained in 3690?

2. How many times can 246 be subtracted from 3690?

3. How many times 246 will produce 3690?

4. If the divisor be 1246 and the dividend 33642, what is the quotient?

5. The factors of a number are 84+23 and 427—259. What is the number?

6. The product of two numbers is 14304, and one of the numbers is 2400÷25. What is the other?

7. What number added to 1863 will amount to 3248?

8. Divide the product of 96 and 48 by their difference.

9. What is the difference between 106 × 127, and 15341 ÷ 29?

10. What is the sum of 4527 — 1039, 250 × 86, and 3025 + 76?

11. The divisor is 16, and the quotient 12624. If the divisor is diminished 4 times, what is the quotient?

12. Of what number is 306 the divisor and quotient?

13. How many pounds of butter at 40 cents a pound are worth as much as 1600 bushels of oats at 75 cents a bushel?

14. If a man gain $638.75 by selling 365 barrels of flour at $9.25 a barrel, at what price did he buy it?

15. If a mechanic receive $1250 a year for labor, and his expenses are $765 a year, how much can he save in 5 years?

16. A planter sold 40 bales of cotton at $68 a bale. With the proceeds he bought 8 horses at $180 each, and with the remainder bought cows at $32 a head. How many cows did he purchase?

17. The multiplier is 48, the product is 166656; if the multiplicand is 1 third as great, what is the product?

18. The divisor is 16, the quotient 12624; if the divisor is 1 fourth as great, what is the quotient?

19. The divisor is 24, and the quotient is 43950; if the divisor is made 6 times as large, what is the quotient?

20. The quotient is 91864; if the divisor is 1 ninth as great, what is the quotient?

21. A grocer bought two kinds of syrup; one for 54 cents a gallon, and the other for 62 cents. What was the *average* cost a gallon?

OPERATION.—(54 cents + 62 cents) ÷ 2 = 58 cents.

The *average* of *two* numbers is *one-half* their sum, the average of *three* numbers is *one-third* their sum, etc.

22. A merchant bought 3 kinds of tea, some at 60 cents, some at 78 cents, and some at 90 cents a pound. What was the average cost a pound?

23. A keeper of a toll bridge received $104 toll on Monday, $97 on Tuesday, $128 on Wednesday, and $99 on Thursday. What was the average daily receipts?

24. Sold 3 city lots for $1500, $2976, and $1895, respectively. What was the average price of each?

25. If a young man receive a salary of $25 a week, and he pays $8.75 for his board, and $4.65 for other expenses, in how many weeks can he pay a debt of $487.20?

26. A man having $4578, paid out all but $1642, in 8 weeks. What was the average amount paid out each week?

27. Bought 140 acres of land for $7560, and sold 86 acres of it, at $75 an acre, and the remainder at cost. What was gained?

28. A father gave his property to his 4 children. To the first he gave $6780, to the second $8200, to the third $1526 more than to the first, and to the fourth $1345 less than to the third. What was the value of his property?

29. The sum of two numbers is 184, and their difference is 42. What are the numbers?

Analysis.—Since 184 is the sum of the numbers, if the difference 42 be subtracted from the sum 184, the remainder 142 will be *twice* the *less* number. 142 ÷ 2 = 71, the less number; and 71 + 42 = 113, the *greater* number.

Or, if the *difference* 42 be added to the *sum* 184, the amount 226, will be *twice* the *greater* number. 226 + 2 = 113, the greater number; and 113 − 42 = 71, the less number.

Proof.—113 + 71 = 184, the sum.

30. The sum of two numbers is 5672, and their difference is 1974. What are the numbers?

31. A man paid $1250 for a horse and carriage, the horse being valued at $190 more than the carriage. What was the value of each?

32. At an election the number of votes cast for two candidates was 3789, and the majority for the successful candidate was 227. How many votes did each receive?

33. Two men are worth $28475, and one is worth $4625 more than the other. How much is each man worth?

34. Bought 228 barrels of flour for $1482, and sold the same for $1938. What was the gain on each barrel?

35. A farmer sold 300 bushels of wheat at $2 a bushel, corn and oats to the amount of $750; with the proceeds he bought 120 head of sheep at $3 a head, one pair of oxen for $90, and 25 acres of land for the remainder. How much did the land cost him an acre?

36. The sales of a merchant are $75000 a year. What is the average per week ? Per month ?

37. How many pounds of sugar at 20 cents a pound must be given for 5 pieces of cotton cloth, containing 44 yards each, at 42 cents a yard ?

38. How many square yards in a floor 10 ft. by 9 ft. ? In a ceiling 12 ft. square ? In a wall 11 ft. by 12 ft. ?

39. A grocer bought 12 barrels of sugar at $16 per barrel, and 17 barrels at $13 per barrel; how much would he gain by selling the whole at $18 per barrel ?

40. A farmer sold 16 cords of wood at $3 a cord, and 40 bushels of wheat at $2 a bushel. He received 15 yards of cloth at $4 a yard, and the remainder in money; how much money did he receive ?

41. If a mechanic receive $1500 a year for his labor, and his expenses are $968, in what time can he save enough to buy 28 acres of land at $133 an acre ?

42. A farmer has 20 acres of meadow, yielding 2 tons of hay per acre; 45 acres of wheat, yielding 14 bushels per acre; 120 acres of corn, yielding 35 bushels per acre. He sells his hay at $6 a ton, his wheat at 95c. a bushel; one-half of his corn he sells at 28c. a bushel, and the remainder he feeds to 100 hogs, which at selling averaged 230 pounds and brought $4.50 per cwt. If first cost of hogs, and his farming expenses were $690, what has he made or lost ?

Find the value of each of the following equations:

43. $384 + 4621 - 872 + (\overline{3176 \div 8) \times 12} = ?$

44. $25 \times 370 - (62 \times 8) + (\overline{335 + 206} \times 45) = ?$

45. $16 \times 3125 - (127 \times 0 + (\overline{972 - 122) \div 50} = ?$

192. The pupil should be required to illustrate the following problems by original examples:

1. Given, several numbers, to find their sum.

2. Given, the sum of several numbers and all of them but one, to find that one.

3. Given, two numbers, to find their difference.

4. Given, the minuend and subtrahend, to find the remainder.

5. Given, the minuend and remainder, to find the subtrahend.

6. Given, the subtrahend and remainder, to find the minuend.

7. Given, two or more numbers, to find their product.

8. Given, the multiplicand and multiplier, to find the product.

9. Given, the product and multiplicand, to find the multiplier.

10. Given, the product and multiplier, to find the multiplicand.

11. Given, the divisor and dividend, to find the quotient.

12. Given, the divisor and quotient, to find the dividend.

13. Given, the dividend and quotient, to find the divisor.

14. Given, the divisor, quotient, and remainder, to find the dividend.

15. Given, the dividend, quotient, and remainder, to find the divisor.

16. Given, the sum and the difference of two numbers, to find the numbers.

PROPERTIES OF NUMBERS.

193. An ***Integral Number*** or ***Integer*** is a number representing whole things (**106**).

Thus, 8, 23, 30 men, 45 pounds are integral numbers.

194. An ***Even Number*** is a number that is exactly divisible by 2.

All numbers whose unit figure is 0, 2, 4, 6, or 8, are *even*.

195. A ***Prime Number*** is a number that has no integral factors except unity and itself.

Thus, 2, 3, 5, 11, 23, etc., are prime numbers.

196. A ***Composite Number*** is a number that has other *integral* factors besides unity and itself.

Thus, 21 is a composite number, since $21 = 7 \times 3$.

197. A ***Factor*** of a number is one of its *equal parts*, or it is one of its exact divisors. (**162.**)

198. A ***Prime Factor*** is a prime number used as a *factor*. (**195.**)

The *prime factors* of a number are also the *prime divisors* of it.

199. An ***Exact Divisor*** of a number is one that will divide that number without a remainder.

Thus, 6 is an exact divisor of 48, and 9 an exact divisor of 72.

1. The *Exact Divisors* of a number are also the *factors* of it.

2. An exact divisor of a number is sometimes called the *measure* of that number.

200. Numbers are ***prime to each other*** when they have no *common integral factors, or divisors*.

Thus, 9 and 14, 16 and 25 are prime to each other.

DIVISIBILITY OF NUMBERS.

201. A number is said to be divisible by another, when there is no remainder after dividing. Any number is *divisible*

1. By 2, if it is an *even* number.

Thus, 20, 24, 36, and 44 are divisible by 2.

2. By 3, if the sum of its digits is divisible by 3.

Thus, 135, 471, and 1134 are divisible by 3.

3. By 4, if its two right-hand figures are ciphers, or express a number divisible by 4.

Thus, 300, 432, and 1548 are divisible by 4.

4. By 5, if it ends with a cipher or 5.

Thus, 30, 45, and 235 are divisible by 5.

5. By 6, if it is an *even* number and divisible by 3.

Thus, 168, 402, and 1314 are divisible by 6.

6. By 8, if its three right-hand figures are ciphers, or express a number divisible by 8.

Thus, 3000, 2728, and 10576 are divisible by 8.

7. By 9, if the sum of its digits is divisible by 9.

Thus, 217683 and 401301 are divisible by 9.

EXERCISES.

202. Find by inspection some of the exact divisors of the following numbers:

1. 1536.
2. 1683.
3. 3348.
4. 6105.
5. 12936.
6. 43560.
7. 32472.
8. 71460.
9. 197200.

FACTORING.

203. *1.* What are the prime numbers from 12 to 36?

2. What are the composite numbers from 12 to 36 ?

3. What are the prime factors of 35 ? 49 ? 60 ?

4. What are the composite factors of 32 ? 48 ? 72 ?

5. What prime factor is common to 21 and 42 ?

6. What composite factors are common to 36 and 72 ?

7. What factors are common to 18 and 30 ? To their *sum ?* To their *difference ?*

204. ***Factoring*** is the resolving of a composite number into its factors, and is performed by division.

When a number is used as a factor *twice*, the product is called the *second power* or *square* of the number. When used as a factor *three* times, the product is called the *third power* or *cube*, etc., etc. Thus, $3 \times 3 = 9$, the *square* of 3 ; $3 \times 3 \times 3 = 27$, the *cube* of 3, etc.

205. An ***Exponent*** is a small figure written at the right of a number, and a little above, to show how many times the number is used as a factor.

Thus, $2^3 = 2 \times 2 \times 2$, and denotes that 2 is used as a factor 3 *times.*

206. PRINCIPLES.—*1. The prime factors of a number, or the product of any two or more of them, are the only exact divisors of that number.*

2. A factor of a number is a factor also of any number of times that number.

3. A factor common to two or more numbers is a factor of their sum ; also, of the difference of any two of them.

4. Every composite number is equal to the product of its prime factors.

WRITTEN EXERCISES.

207. TO resolve a composite number into its prime factors.

1. What are the prime factors of 252?

OPERATION.

2	252
2	126
3	63
3	21
	7

ANALYSIS.—Since the given number is ever, divide by the least prime factor 2, and the quotient, also by 2.

Next divide by the prime number 3 and 3 successively, obtaining 7 for the last quotient, which being prime, the division can be carried no further. Hence the divisors 2, 2, 3, 3, and the last quotient 7 are all the prime factors, or divisors, of 252, and may be written 7, 3^2, 2^2.

PROOF.—$7 \times 3 \times 3 \times 2 \times 2 = 252$.

2. What are the prime factors of 126? Of 225? Of 154?

3. Find the prime factors of 200; of 392; of 440.

RULE.—*Divide the given number by any prime factor of it, and the resulting quotient by another, and so continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

PROOF.—*The product of all the prime factors is equal to the given number.* (PRIN. 4.)

Resolve the following numbers into their prime factors:

4. Of 625.
5. Of 504.
6. Of 704.
7. Of 539.
8. Of 945.
9. Of 1050.
10. Of 1140.
11. Of 1155.
12. Of 1728.
13. Of 2205.
14. 1155.
15. 2934.
16. 6300.
17. 2205.
18. 9375.
19. 12673.
20. 10010.
21. 28665.
22. 31570.
23. 29925.

COMMON DIVISORS.

208. A ***Common Divisor*** of two or more numbers is a *common factor* of each of them.

209. The ***Greatest Common Divisor*** of two or more numbers is the *greatest common factor*, and is the product of *all* the common *prime* factors, and no more.

210. PRINCIPLES.—*1. The only exact divisors of a number are its prime factors, or the product of two or more of them.*

2. An exact divisor divides any number of times its dividend.

3. A common divisor of two or more numbers will divide their sum, and also the difference of any two of them.

4. The greatest common divisor of two or more numbers is the product of all their common prime factors.

WRITTEN EXERCISES.

211. When the numbers can be readily factored.

1. What is the greatest common divisor of 42, 63, and 126?

1ST OPERATION.

$42 = 7 \times 3 \times 2$

$63 = 7 \times 3 \times 3$

$126 = 7 \times 3 \times 6$

ANALYSIS.—By factoring the given numbers, the prime factors common to *all* of them are 7 and 3. Hence $7 \times 3 = 21$ is the greatest common divisor of 42, 63, and 126. (PRIN. 4.)

2D OPERATION.

$$\begin{array}{r|rrr} 3 & 42 & 63 & 126 \\ \hline 7 & 14 & 21 & 42 \\ \hline & 2 & 3 & 6 \end{array}$$

ANALYSIS.—Since the given numbers are exactly divisible by 3, and the resulting quotients by 7, they are also divisible by 7×3, or 21. (PRIN. 1.)

If there wore other factors of the greatest common divisor, then the quotients 2, 3, and 6 would be exactly divisible by them.

Find the greatest common divisor

2. Of 42 and 112.

3. Of 96 and 544.

4. Of 679 and 1869.

5. Of 1080 and 189.

6. Of 40, 75, and 100.

7. Of 72, 126, and 216.

RULE.—I. *Separate the numbers into their prime factors and find the product of all that are common.* Or,

I. *Write the numbers in a line, and divide by any prime factor common to all the numbers.*

II. *Divide the quotients in like manner, and so continue the division till all the quotients are prime to each other.*

III. *The product of all the divisors will be the greatest common divisor.* (PRIN. 4.)

What is the greatest common divisor

8. Of 144 and 720?

9. Of 308 and 506?

10. Of 126, 210, and 252?

11. Of 72, 96, 126, and 384?

212. When the numbers cannot be readily factored.

1. Find the greatest common divisor of 507 and 1207.

OPERATION.

		1207
527	2	1054
459	3	153
68	2	136
68	4	17

ANALYSIS.—Draw two vertical lines, and place the larger number on the right, and the smaller on the left, one line lower down. Divide 1207 by 527, and write the quotient 2 between the vertical lines, the product, 1054, under the greater uumber, and the remainder 153, below.

Next, divide 527 by this remainder 153, writing the quotient 3 between the verticals, the product 459, on the left, and the remainder 68, below.

Again, divide the last divisor 153, by 68, and write the product and remainder in the same order as before.

Finally, dividing the last divisor 68, by the last remainder 17, there is no remainder. Hence 17, the last divisor, is the greatest common divisor of 537 and 1207.

In like manner find the greatest common divisor

2. Of 825 and 1372.

3. Of 1008 and 1036.

RULE—I. *Draw two vertical lines, and write the two numbers, one on each side, the greater number one line above the less.*

II. *Divide the greater number by the less, writing the quotient between the verticals, the product under the dividend, and the remainder below.*

III. *Divide the less number by the remainder, the last divisor by the last remainder, and so on, till nothing remains. The last divisor is the greatest common divisor sought.*

IV. *If more than two numbers are given, first find the greatest common divisor of two of them, and then of this divisor and one of the remaining numbers, and so on to the last; the last common divisor found is the greatest common divisor of the given numbers.*

4. What is the greatest number that will divide 2041 and 8476? 3281 and 10778?

5. What is the greatest number that will divide 216, 360, and 432? 141, 799, and 940?

6. B has $620, C $1116, and D $1488, with which they buy horses, at highest price per head that allows each man to invest all his money. How many horses does each buy?

7. A merchant has 15292 bushels of wheat, 1520 bushels of corn, and 504 bushels of beans, which he wishes to ship in the fewest bags of equal size that will exactly hold either kind of grain. How many bags will it take?

MULTIPLES.

213. *1.* What numbers between 5 and 30 are exactly divisible by 4? By 6? 7? 8? 9?

2. What numbers less than 40 are exactly divisible by 7?

3. What prime factors are common to 6, and 5 times 6?

4. Name some numbers exactly divisible by 4 and 6.

5. By what three prime numbers can 42 be divided?

6. Name some numbers of which 3 and 4 are factors.

7. Find the least number exactly divisible by 3, 4, and 5.

214. A ***Multiple*** of a number is a number exactly divisible by the given number; or, it is any product or dividend of which the given number is a *factor*.

Thus, 15, 20, and 25 are multiples of 5.

215. A ***Common Multiple*** of two or more given numbers is a number exactly divisible by each of them.

216. The ***Least Common Multiple*** of two or more given numbers is the least number exactly divisible by each of them.

217. PRINCIPLES.—*1. A multiple of a number contains each of the prime factors of that number.*

2. A common multiple of two or more numbers contains each of the prime factors of those numbers. Hence,

3. The least common multiple of two or more numbers is the least number that contains each of the prime factors of those numbers.

4. A common multiple of two or more numbers may be found by multiplying the given numbers together.

WRITTEN EXERCISES.

218. To find the least common multiple.

1. Find the least common multiple of 30, 42, and 66.

OPERATION.

$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$
$$66 = 2 \times 3 \times 11$$
$$2 \times 3 \times 11 \times 7 \times 5 = 2310$$

ANALYSIS.—The least common multiple cannot be less than the largest number 66, since it must contain 66; hence it must contain all the prime factors of 66, 2, 3, and 11. (PRIN. 1.)

But the least common multiple of 66 must also contain all the prime factors of each of the other numbers, and since the prime factors 2 and 3 of 66 are common also to 42 and 30 omit them, and annex the factors 7 and 5 to those of 66, and the series 2, 3, 11, 7, and 5 are all the prime factors of the given numbers, and their product $2 \times 3 \times 11 \times 7 \times 5 = 2310$, is the least common multiple of the given numbers. (PRIN. 3.)

2. Find the least common multiple of 24, 42, and 17.

3. Find the least common multiple of 8, 12, 20, and 30.

4. Find the least common multiple of 8, 12, 36, and 72.

RULE.—I. *Resolve each of the given numbers into its prime factors.*

II. *Multiply together all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

Find the least common multiple

5. Of 72 and 48.
6. Of 52 and 78.
7. Of 144 and 180.
8. Of 324 and 360.
9. Of 225 and 375.
10. Of 12, 15 and 42.
11. Of 21, 35 and 56.
12. Of 16, 40 and 96.
13. Of 84, 100 and 224.
14. Of 16, 20, 48 and 72.

15. What is the least number of marbles that can be divided equally among 12, 18, and 24 boys?

16. What is the smallest sum of money for which I can purchase an exact number of books at $3, or $4, or $5, or $6 each?

17. What is the product of the least common multiple of 12, 16, 24, and 32, multiplied by their greatest common divisor?

18. Divide the least common multiple of 7, 42, 6, 9, and 10, by the greatest common divisor of 110, 140, and 680.

19. What is the least number of acres in a farm that can be exactly divided into lots of 10 acres, 14 acres, 16 acres and 20 acres each?

CANCELLATION.

219. *1.* Divide 1 third of 48 by 1 third of 12.

2. Divide 1 fourth of 48 by 1 fourth of 12.

3. What factors are common to 48 and 12?

4. Rejecting the factors common to 48 and 12, what factors of each will remain?

5. What is the quotient of 16 × 3 divided by 4 × 3?

6. What is the quotient of 12 × 4 divided by 3 × 4?

7. What is the quotient of 2 × 5 × 3 divided by 2 × 5?

220. ***Cancellation*** is the process of abridging operations in division by rejecting equal factors from both dividend and divisor.

221. PRINCIPLES.—1. *Rejecting a factor from any number divides the number by that factor.*

2. *Rejecting equal factors from both dividend and divisor does not change the quotient.*

WRITTEN EXERCISES.

222. *1.* What is the quotient of 9 times 14 divided by 6 times 7?

OPERATION.

$$\frac{9\times 14}{6\times 7}=\frac{3\times\cancel{3}\times\cancel{2}\times\cancel{7}}{\cancel{2}\times\cancel{3}\times\cancel{7}}=\frac{3}{1}=3$$

ANALYSIS.—Write the numbers as in division, the dividend above and the divisor below the line. Resolve the numbers into their factors, the dividend being composed of 3 × 3 × 2 × 7, and the divisor of 2 × 3 × 7. Cancelling equal factors from both dividend and divisor, which is the same as dividing by those factors, there is left the factor 3 in the dividend. Hence the quotient is 3.

2. Divide 12 × 8 × 6 by 8 × 4 × 3.

OPERATION.

$$\frac{\overset{3}{\cancel{12}}\times\cancel{8}\times\overset{2}{\cancel{6}}}{\cancel{8}\times\cancel{4}\times\cancel{3}}=\frac{3\times 2}{1}=6$$

ANALYSIS.—In this example first strike out the factor 8, common to the dividend and divisor. Next cancel the factor 4 from 4 and 12, leaving the factor 3 in the dividend. In like manner cancel the factor 3 from 3 and 6, leaving the factor 2 in the dividend. Hence the quotient is 3 × 2 = 6.

By many it is thought more convenient to write the factors of the *dividend* on the *right* of a *vertical* line, and the factors of the *divisor* on the *left*.

3. Divide the product of 25, 18, 4, and 3 by the product of 7, 6, 5, and 3.

OPERATION.

$$\frac{\overset{5}{\cancel{25}}\times\overset{3}{\cancel{18}}\times 4\times\cancel{3}}{7\times\cancel{6}\times\cancel{5}\times\cancel{3}}=\frac{5\times 3\times 4}{7}=\frac{60}{7}=8\tfrac{4}{7}.\ \text{Or,}\quad \begin{array}{r|l} 7 & \cancel{25}\ ^{5} \\ \cancel{6} & \cancel{18}\ ^{3} \\ \cancel{5} & 4 \\ \cancel{3} & \cancel{3} \\ \hline 7 & 60 \\ \cline{2-2} & 8\tfrac{4}{7} \end{array}$$

In like manner,

4. Divide 13 × 7 × 5 × 3 by 3 × 5 × 7.

5. Divide 42 × 18 × 6 × 4 by 36 × 21 × 6.

RULE.—I. *Cancel all the factors common to both dividend and divisor.*

II. *Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

When a factor equal to the number itself is canceled, the unit 1 remains, since a number divided by itself gives a quotient of 1.

If the 1 occur in the dividend, it must be *retained;* if in the divisor, it need not be regarded.

6. What is the quotient of $21 \times 8 \times 60 \times 8 \times 6$ divided by $7 \times 12 \times 3 \times 8 \times 3$?

7. Divide the product of 200 multiplied by 60, by the product of 50 multiplied by 48.

8. How many barrels of flour at $9 a barrel must be given for 27 yards of cloth at $4 a yard?

9. How many bushels of apples at 75 cents a bushel will pay for 85 pounds of coffee at 30 cents a pound?

10. A farmer exchanged 45 bushels of potatoes worth 40 cents a bushel for 18 pounds of tea. What was the tea worth a pound?

11. From 5 pieces of cloth, each piece containing 24 yards worth $3 a yard, how many suits can be made worth $18 a suit?

12. How many bales of cloth, each containing 40 pieces, and each piece containing 36 yards, worth $3 a yard, must be given for 120 government bonds worth $108 each?

13. How many barrels of potatoes, each containing 3 bushels, worth 30 cents a bushel, will pay for 12 boxes of soap, each containing 51 pounds, worth 10 cents a pound?

REVIEW AND TEST EXAMPLES

223. *1.* The area of the United States is 1,996,487 square miles; of the Territories, 1,042,000, and of Alaska, 577,390; how much greater is the area of the States than of the Territories and Alaska?

2. The population of London in 1870 was 3,250,000; of New York, 944,129; and of Brooklyn, 396,300. How much greater was the population of London than of New York and Brooklyn?

3. The total length of railroads in Massachusetts Jan. 1st, 1864, was 1941 miles, and the total cost of these roads and equipments up to the same date was $66,809,802. How much had been expended per mile?

4. Texas has an area of 274,356 square miles, Massachusetts 7800 square miles. How many States, each as large as Massachusetts, could be formed from Texas?

5. If Texas were divided into 5 equal States, how much larger than New York, which contains 47000 square miles, would each of them be?

6. The foreign immigration to this country for ten years, commencing with 1861 and ending with 1870 inclusive, was as follows:

1861	65,539
1862	76,306
1863	156,844
1864	182,296
1865	196,352
1866	233,418
1867	242,731
1868	213,606
1869	358,989
1870	211,190

What was the total, and what the average per year?

Measures.—Refer to the tables, Pages 61 to 81, so far as is necessary, in solving the following examples:

7. In 230 pints, how many quarts? How many gallons?

8. How many seconds in 10 minutes and 22 seconds? In 25 minutes and 45 seconds?

9. How many rods in 20 miles? In 16 miles 15 rods? In 35 miles 56 rods? In ½ mile? In ¾?

10. How many minutes in 1½ days? In 2¾ days? How many hours in 1 week? In 1 week 3 days?

11. What part of a bushel is a peck? Of a peck, is a quart? Of a bushel, is a quart?

12. A merchant bought a keg of nails for $7.50. For what must he sell the same, to gain half a cent a pound?

13. A gross of pens costs 40 cents. If sold at the rate of 2 for a cent, what will be the profit on a gross?

14. Bought a barrel of eggs containing 100 dozen, at the rate of 4 for 5 cents. What did they all cost? At the same rate, how many can be bought for one dollar? How many for $2½? For $5?

15. A farmer has an orchard containing 577 trees; each tree produces 3 barrels of apples, each containing 2½ bushels. He sells them for 50 cents a bushel. What does he receive for his apples, if it costs him $50 freight and $27 commission to sell them?

16. Draw a square, each side of which shall be 1 inch, and divide it into 4 equal squares. What part of the whole is one square? Two squares? Three squares?

17. Into how many squares, each measuring 1 inch on a side, can you divide a square measuring 3 inches on a side? Measuring 4 inches on a side?

18. To draw by a scale twice as large, is to make each line twice as long. Draw a square by a scale twice as large as that in example 16. How long is one side of this square? How far around it? Into how many squares, each the size of that in example 16, can it be divided?

19. If a square is drawn on a scale twice as large as another, how many times as much surface does it represent?

20. If a square is drawn on a scale three times as large as another, how many times as much surface does it represent? Draw squares to illustrate this.

21. In like manner, after drawing a square, draw others on a scale 4 times as large. 5 times as large, etc.

22. How many solid feet in a wall 20 feet long, 16 feet high, and 2 feet thick?

23. How many small squares, each containing 6 sq. in., are equal to one large one, 3 feet square?

24. If 3 persons can stand on 1 square yard of a floor, how many could stand in a room 10 yards long and 7 yards wide?

25. What is the distance around a rectangular field, whose length is twice its breadth, if one end measures 35 rods?

26. How many cubic feet in a box 6 feet long, 4 feet wide, and 3 feet high?

27. A grocer bought 60 gallons of milk at 12 cents a gallon, and sold it at 5 cents a quart. What did he gain?

28. What will be the cost of 12 boards, each measuring 12 feet long and 1 foot wide, at two cents a square foot?

29. What is the distance around a public square, which measures 12 rods on a side?

30. How many acres in a piece of land 60 rd. square?

31. What will it cost to remove an embankment 120 ft. long, 64 ft. wide, and 9 feet high, at $.54 a cubic yard?

32. At $3.80 a cord, what will be the value of the wood that can be piled under a shed 40 feet long, 20 feet wide, and 15 feet high?

33. What will be the cost of a block of granite, 7 ft. long, 4 ft. wide, and 3 ft. thick, at $.25 a cu. ft.?

34. If a drover buy 24 lambs for $30, at how much must he sell them a head to gain $18?

35. A laborer receives $2.50 a day wages, and spends $1.15 a day, including Sunday, for his support. How much does he save in 1 week? In 4 weeks? In 10 weeks?

36. Two vessels are 150 miles apart, and sail directly toward each other, one at the rate of 8 miles an hour, and the other at the rate of 12 miles an hour; how many hours before they will meet?

37. If 5 barrels of cider are worth $20, how many tons of hay, at $12 a ton, will 9 barrels of cider buy?

38. If a stage-coach run 72 miles in 12 hours, in how many hours will it run 96 miles?

39. How many dozen of eggs at 20 cents a dozen will pay for 18 pounds of sugar at 10 cents a pound?

40. A farmer sold 6 loads of apples, each containing 14 barrels, and each barrel 3 bushels, at $.50 a bushel. He received in payment 9 barrels of sugar, each weighing 210 pounds. What was the sugar worth a pound?

41. At the rate of 12 miles in 3 hours, how far would a man travel in 10 hours?

42. How many gallons of molasses, at 45 cents a gallon, will pay for 120 pounds of cheese, at 9 cents a pound?

43. What is the smallest sum of money for which I can purchase an exact number of sheep at $8, or cows at $28, or oxen at $54, or horses at $162 each?

44. Bought a farm for $22360, and after expending $1720 in improvements upon it, I sold one-half of it for $15480, at $18 per acre. How many acres of land did I purchase, and at what price per acre?

45. The sum of two numbers is 2487, and the greater is 553 more than the less. What are the numbers?

46. Having money to invest I buy 2 farms worth $4550 each, 20 shares railroad stock at $106 per share, and have $250 left. How much had I at first?

47. A grocer gave 153 barrels of flour worth $9 a barrel for 81 barrels of sugar containing 170 pounds each. What was the cost of the sugar per pound?

48. If I pay $1.37 a bushel for wheat, $.95 for rye, and $.73 a bushel for corn, how much, of each an equal number of bushels, can I purchase for $70.15?

49. A house and lot valued at $1200, and 6 horses at $95 each, were exchanged for 30 acres of land. At how much was the land valued per acre?

50. Two travelers set out at the same time from places 396 miles distant from each other, and travel towards each other till they meet, when it appears that one has come 27 miles a day, and the other 31 miles a day; how many days were they on the journey?

51. If 2, 2, 5, and 7 are four factors of 1680, what is the other factor?

52. A grocer bought 12 barrels of sugar at $16 per barrel, and 17 barrels at $13 per barrel; what would he gain by selling the whole at $18 per barrel?

53. A man bought 750 acres of land at $20 an acre; he sold at one time 325 acres at $23 per acre, at another time 260 acres at $25 per acre. At what price per acre must he sell the remainder to gain on his purchase $1945?

54. Multiply 64 by 7 times 31 and divide the product by 8 times 56; multiply this quotient by 15 times 88 and divide the product by 55; multiply this quotient by 13 and divide the product by 4 times 6.

55. The population of eight of the largest cities of France in 1866 was as follows:

Cities.	Pop.	Cities.	Pop.
Paris	1,825,274	Lille	154,749
Lyons	323,954	Toulouse	126,936
Marseilles	300,131	Nantes	111,956
Bordeaux	194,241	Rouen	100,671

How much did the population of Paris exceed that of all the other cities?

56. Paid $9285 for a farm; ¼ as much for a house. What did both cost?

57. If 58 pounds of butter cost $14.50, what will 63 pounds cost?

58. Sound moves 1090 feet per second. If I see the flash of a gun and hear the report in 5 seconds, how far am I from the gun?

59. The cotton crop of several States in 1870, the number of acres cultivated, and bales produced, were as follows:

State.	Acres.	Bales.
North Carolina	451,714	170,000
North Carolina	601,764	220,000
Georgia	1,330,491	495,000
Florida	140,909	50,000
Alabama	1,437,272	510,000
Mississippi	1,544,512	725,000
Louisiana	920,700	495,000
Texas	900,937	465,000
Arkansas	711,734	375,000
Tennessee	526,184	215,000

What was the total number of acres cultivated, and bales produced?

60. What number, divided by 453, will give the quotient 307, and the remainder 109?

61. A coal dealer paid $965 for some coal. He sold 160 tons for $5 a ton, when the remainder stood him in but $3 a ton. How many tons did he buy?

62. How many pounds of sugar at 20 cents a pound must be given for 5 pieces of cotton cloth, containing 44 yards each, at 42 cents a yard?

63. A farmer sold 300 bushels of wheat at $2 a bushel, corn and oats to the amount of $750; with the proceeds he bought 120 head of sheep at $3 a head, one pair of oxen for $90, and 25 acres of land for the remainder. What did the land cost him per acre?

64. A grocer bought 25 barrels of sugar at $25 a barrel, and 34 barrels at $22 a barrel. How much would he gain by selling the whole at $27 a barrel?

65. Iron was discovered in Greece by the burning of Mount Ida, B.C. 1406; and the electro-magnetic telegraph was invented by Morse, A.D. 1832. What period of time elapsed between the two events?

66. If a man receive $1200 salary, and pay $364 for board, $275 for clothing, $150 for books, and $187 for other expenses, how much can he save in 5 years?

67. A speculator gained $5760, and afterwards lost $2746; at another time he gained $3575, and then lost $4632. How much did his gains exceed his losses?

68. A mechanic earns $75 a month, but his necessary expenses are $50 a month. How long will it take him to pay for a farm of 60 acres worth $40 an acre?

69. A man worth $12750 received a legacy of $8000; he spent $456 in traveling, and gave $1000 to a college. How much money had he left?

70. A farmer bought a lot of sheep and hogs, of each an equal number, for $1276. He gave $4 a head for the sheep, and $7 a head for the hogs; what was the whole number purchased, and what was the difference in the total cost of each?

71. The population of ten of the largest cities in the United States was as follows:

Cities.	1870.	1860.	Cities.	1870.	1860.
New York	944,129	805,658	Baltimore	267,354	212,418
Philadelphia	674,022	565,529	Boston	250,526	177,840
Brooklyn	396,300	266,661	Cincinnati	216,239	101,044
St. Louis	310,864	160,773	New Orleans	191,322	168,675
Chicago	298,983	109,260	San Francisco	149,482	56,802

What was the total increase of these ten cities in ten years?

72. What was the average increase per year?

73. What was the increase of each city during the time?

74. How much larger was New York in 1870 than each of the other cities?

75. The area of the Pacific Ocean is 65,630,000 square miles; of the Atlantic Ocean, 34,780,000; of the Indian Ocean, 30,590,000. Which is the larger, the Pacific or the other two combined; and how much?

76. What is the area of a floor 27 feet long and 18 feet wide? What will it cost to cover the floor with carpeting a yard wide, at $2.25 a yard?

CHICAGO, *June 20, 1877.*

77. Mr. James E. Rice,

Bought of James Brown.

67	pairs of calf boots............	@	$3.25
75	" " gaiters..............	"	1.12
35	" " slippers.............	"	.70
50	" " rubbers.............	"	1.04
20	" " arctics..............	"	1.10
		Amt. of bill.	$400.25

78. If I sell a lady a set of furs for $45.50; 10 yd. of silk at $2.50 per yd.; a bonnet for $7.50; a pair of gloves for $1.75; and an umbrella for $3.50, what is the amount of the bill?

79. Make out bill for the following: Jas. Smith bought of Johnson & Co., 25 lb. of coffee sugar at 11¢; 5 lb. of tea at 75¢; a box of crackers containing 25 lb. at 6¢; a box of soap for $1; and 3 lb. of coffee at 35¢.

80. If you go to the store of Jones & Co., and buy 5 yd. of cloth at $1.25 a yd.; 32 yd. of flannel at 60 cents a yard; 40 yd. of muslin at 11 cents; 1 pair of gloves for $2; 20 yd. of carpet at $1.10, what is the amount of your bill?

FRACTIONS.

224. A ***Fraction*** is one or more of the *equal parts* of a unit, or of anything regarded as a *whole.*

Review from Page 54 to Page 57 inclusive.

225. A ***Fractional Unit*** is *one* of the equal parts of a unit, or of anything regarded as a whole.

Fractions are classified as *Proper* or *Improper.*

226. A ***Proper Fraction*** is a fraction whose numerator is less than its denominator. Its *value* is *less* than a unit. Thus, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{11}{14}$ are proper fractions.

227. An ***Improper Fraction*** is a fraction whose numerator equals or exceeds its denominator. Its *value* is *equal to,* or *greater* than a unit. Thus, $\frac{6}{6}$, $\frac{10}{4}$, and $\frac{21}{9}$ are improper fractions.

228. ***The Value*** of a fraction is the quotient of its numerator divided by its denominator; thus $\frac{9}{3} = 3$.

229. Since a fraction may indicate division, all changes in the *terms* of a fraction will affect the *value* of that fraction according to the laws of division (**187**), and may be embraced in the following

GENERAL LAW.

230. *A change in the NUMERATOR produces a LIKE change in the value of the fraction; but a change in the DENOMINATOR produces an OPPOSITE change in the value of the fraction.* (See Page 137.)

REDUCTION.

231. ***Reduction of Fractions*** is the process of changing their *form* without changing their *value.*

232. To reduce fractions to higher terms.

A fraction is reduced to *Higher Terms,* when the numerator and denominator are expressed in larger numbers of the same comparative value.

233. PRINCIPLE.—*Multiplying both terms of a fraction by the same number does not change the value of the fraction.*

WRITTEN EXERCISES.

1. Reduce $\frac{5}{8}$ to sixteenths.

OPERATION.

$16 \div 8 = 2$

$\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$

ANALYSIS. — Since the given denominator 8 is contained in the required denominator 2 times, multiply both terms of the fraction by 2. (PRIN.) Hence $\frac{5}{8} = \frac{10}{16}$.

2. Change $\frac{5}{6}$ to a fraction whose denominator is 30.

3. Change $\frac{7}{9}$ to a fraction having 45 for its denominator.

4. Reduce $\frac{3}{8}$ to a fraction having 40 for its denominator.

RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply the terms of the fraction by the quotient.*

Reduce	Change
5. $\frac{4}{5}$ to thirty-fifths.	*8.* $\frac{11}{15}$ to seventy-fifths.
6. $\frac{5}{7}$ to sixty-thirds.	*9.* $\frac{15}{18}$ to 108ths.
7. $\frac{7}{12}$ to eighty-fourths.	*10.* $\frac{21}{32}$ to 256ths.

11. Change $\frac{7}{12}$ to a fraction, whose denominator is 120.

234. To reduce fractions to lower terms.

A fraction is reduced to its *Lowest Terms*, when its numerator and denominator are *prime to each other*.

235. PRINCIPLES.—*Dividing both terms of a fraction by the same number does not change its value.*

1. How many *fourths* are equal to $\frac{9}{12}$? $\frac{12}{16}$? $\frac{10}{20}$?

2. Reduce $\frac{9}{15}$ to fifths; $\frac{15}{24}$ to eighths.

3. What are the factors common to the terms of $\frac{12}{18}$? $\frac{20}{30}$?

4. Change $\frac{15}{30}$ to a fraction having lower terms; $\frac{18}{27}$; $\frac{16}{32}$.

5. Change $\frac{12}{36}$ to a fraction having a denominator $\frac{1}{3}$ as great; $\frac{1}{4}$ as great; $\frac{1}{2}$ as great.

WRITTEN EXERCISES.

1. Reduce $\frac{36}{48}$ to its lowest terms.

OPERATION.

$\frac{36 \div 4}{48 \div 4} = \frac{9}{12}$; $\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$

Or,

$\frac{36 \div 12}{48 \div 12} = \frac{3}{4}$

ANALYSIS.—Dividing both terms of the given fraction $\frac{36}{48}$ by 4 the result is $\frac{9}{12}$. (PRIN.) Again dividing both terms of $\frac{9}{12}$ by 3 the result is $\frac{3}{4}$. Since the terms of $\frac{3}{4}$ are *prime to each other*, the lowest terms of $\frac{36}{48}$ are $\frac{3}{4}$.

The same result may be obtained more directly by dividing the terms of the given fraction by their greatest common divisor 12.

RULE.—*Reject all factors common to the terms of the given fraction.* Or,

Divide the terms of the given fraction by their greatest common divisor.

Reduce to their lowest terms

2. $\frac{96}{100}$.

3. $\frac{63}{81}$.

4. $\frac{72}{108}$.

5. $\frac{144}{156}$.

6. $\frac{45}{315}$.

7. $\frac{98}{112}$.

8. $\frac{32}{256}$; $\frac{126}{162}$.

9. $\frac{324}{486}$; $\frac{292}{657}$.

10. $\frac{420}{2310}$; $\frac{5400}{7200}$.

236. To reduce an integer or a mixed number to an improper fraction. (Review Pages 56 and 57.,

WRITTEN EXERCISES.

1. Reduce $8\frac{4}{5}$ to fifths.

OPERATION.

$8\frac{4}{5}$
5
40 fifths, and
$\frac{40}{5}+\frac{4}{5}=\frac{44}{5}$

ANALYSIS.—Since 1 is equal to 5 fifths, 8 is equal to 8 times 5 fifths, or 40 fifths, to which add 4 fifths, and the result is 44 fifths.

An *integer* is reduced to a fractional *form* by writing 1 under it for a denominator; thus, $7=\frac{7}{1}$; $15=\frac{15}{1}$.

Reduce

2. 48 to eighths.
3. $156\frac{5}{6}$ to sixths.
4. 124 to ninths.

Change

5. $95\frac{3}{10}$ to tenths.
6. 72 to fourteenths.
7. $216\frac{9}{25}$ to twenty-fifths.

RULE.—*Multiply the integer by the required denominator, and if there be a fraction, add its numerator to this product, and under the result write the denominator.*

Reduce to improper fractions,

8. $75\frac{9}{13}$.
9. $224\frac{5}{18}$.
10. $307\frac{18}{25}$.
11. $176\frac{14}{90}$.
12. $97\frac{7}{120}$.
13. $86\frac{5}{17}$.
14. $450\frac{12}{20}$.
15. $261\frac{7}{80}$.
16. $176\frac{27}{45}$.
17. $304\frac{72}{115}$.
18. $1227\frac{13}{152}$.
19. $2475\frac{24}{30}$.
20. $1462\frac{27}{45}$.
21. $1306\frac{9}{75}$.
22. $2500\frac{3}{64}$.
23. $4006\frac{121}{240}$.

237. To reduce an improper fraction to an integer, or mixed number.

WRITTEN EXERCISES.

1. Reduce $\frac{147}{9}$ to a mixed number.

OPERATION.

$\frac{147}{9}=147\div9=16\frac{1}{3}$

ANALYSIS.—Since 9 ninths equal 1, 147 ninths equal as many times 1 as 9 ninths is contained times in 147 ninths, which is $16\frac{3}{9}$ times. Hence $\frac{147}{9}=16\frac{1}{3}$.

Rule.—*Divide the numerator by the denominator.*

Reduce to integers or mixed numbers,

2. $\frac{651}{7}$.
3. $\frac{247}{12}$.
4. $\frac{476}{17}$.
5. $\frac{1245}{22}$.
6. $\frac{1345}{24}$.
7. $\frac{4407}{113}$.
8. $\frac{4483}{35}$.
9. $\frac{5919}{59}$.
10. $\frac{2184}{42}$.
11. $\frac{7672}{112}$.
12. $\frac{17007}{33}$.
13. $\frac{32460}{48}$.

238. To reduce fractions to equivalent fractions having a common denominator.

1. How many *fourths* in 1 ? In $\frac{1}{2}$?

2. How many *sixths* in 1 ? In $\frac{1}{2}$? In $\frac{1}{3}$?

3. How many *eighths* in 1 ? In $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{4}$?

4. How many *ninths* in 1 ? In $\frac{1}{3}$? In $\frac{2}{3}$?

5. Express $\frac{1}{2}$ and $\frac{1}{3}$ each as twelfths.

6. Express $\frac{1}{3}$ and $\frac{1}{5}$ each as fifteenths.

7. Change $\frac{2}{3}$ and $\frac{3}{4}$ to fractions having the same denominator.

8. Change $\frac{1}{2}$ and $\frac{5}{6}$ to fractions having the same denominator.

9. Multiply both terms of $\frac{4}{5}$ by 3 and explain why the value of the fraction is not changed.

10. What is a multiple of 2 ? Of 3 ? Of 4 ? Of 5 ?

11. What is a common multiple of 2 and 3 ?

12. What is the least common multiple of 2, 5, and 10 ?

13. What is a common multiple of the denominators of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$?

14. What is the least common multiple of the denominators of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$?

15. Find a common multiple of 3, 4, and 6.

16. Name two common multiples of 4 and 8.

17. What is the least common multiple of 2, 4, and 6 ?

18. Change $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having the same fractional unit.

19. Change $\frac{2}{3}$ and $\frac{3}{4}$ to fractions having the same denominator.

20. Reduce $\frac{2}{3}$ and $\frac{1}{9}$ to eighteenths; to twenty-sevenths.

239. Principles.—*1. A common denominator of two or more fractions is a common multiple of their denominators.*

2. The least common denominator of two or more fractions is the least common multiple of their denominators.

WRITTEN EXERCISES.

1. Reduce $\frac{3}{4}$ and $\frac{5}{6}$ to equivalent fractions having a common denominator.

OPERATION.

$$4 \times 6 = 24$$
$$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$
$$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

Analysis.—The product of the denominators 4 and 6 is 24, a common denominator. But since the value of the fractions must not be changed, each numerator must be multiplied by the same number that its denominator is multiplied by.

Hence, multiplying the terms of $\frac{3}{4}$ by 6, the result is $\frac{18}{24}$, and of $\frac{5}{6}$ by 4, the result is $\frac{20}{24}$.

Reduce to equivalent fractions having a common denominator,

2. $\frac{2}{3}$ and $\frac{6}{7}$.
3. $\frac{3}{8}$ and $\frac{5}{12}$.
4. $\frac{1}{6}$ and $\frac{7}{10}$.
5. $\frac{10}{15}$ and $\frac{11}{24}$.
6. $\frac{3}{7}$ and $\frac{5}{9}$.
7. $\frac{7}{12}$ and $\frac{4}{5}$.
8. $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{5}{6}$.
9. $\frac{1}{3}$, $\frac{3}{5}$ and $\frac{4}{7}$.
10. $\frac{4}{5}$, $\frac{1}{6}$ and $\frac{5}{8}$.
11. $\frac{11}{12}$, $\frac{3}{14}$ and $\frac{5}{16}$.
12. $\frac{3}{7}$, $\frac{5}{8}$ and $\frac{2}{3}$.
13. $\frac{7}{16}$, $\frac{2}{3}$ and $\frac{1}{4}$.
14. $\frac{3}{2}$, $\frac{3}{4}$ and $\frac{7}{9}$.
15. $\frac{7}{10}$, $\frac{5}{4}$ and $\frac{5}{6}$.
16. $\frac{7}{12}$, $\frac{1}{2}$, $\frac{4}{5}$ and $\frac{2}{3}$.
17. $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{7}$ and $\frac{8}{9}$.
18. $\frac{9}{10}$, $\frac{5}{7}$ and $\frac{1}{2}$.
19. $\frac{1}{6}$, $\frac{2}{9}$ and $\frac{2}{15}$.

20. Change $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{9}$ to equivalent fractions having the least common denominator.

OPERATION.

$$36 \div 3 = 12; \quad \frac{2 \times 12}{3 \times 12} = \frac{24}{36}$$
$$36 \div 4 = 9; \quad \frac{3 \times 9}{4 \times 9} = \frac{27}{36}$$
$$36 \div 9 = 4; \quad \frac{5 \times 4}{9 \times 4} = \frac{20}{36}$$

ANALYSIS.—The least common multiple of the given denominators is 36, which is their least common denominator.

Dividing this least common denominator by each of the given denominators, we obtain the quotients 12, 9 and 4 respectively.

Multiplying each term of $\frac{2}{3}$ by 12, each term of $\frac{3}{4}$ by 9, and each term of $\frac{5}{9}$ by 4, we change them to equivalent fractions having the least common denominator.

RULE.—*1.* To find a *common* denominator of two or more fractions.

Multiply the terms of each fraction by the denominators of all the other fractions.

2. To find their *least common* denominator.

I. *Find the least common multiple of the denominators of the given fractions for their least common denominator.*

II. *Divide this common denominator by each of the given denominators, and multiply the terms of each fraction by the corresponding quotient. The results are equivalent fractions having the least common denominator.*

Reduce to their least common denominator,

21. $\frac{1}{7}$, $\frac{5}{4}$ and $\frac{12}{24}$.
22. $\frac{3}{4}$, $\frac{7}{10}$ and $\frac{4}{5}$.
23. $\frac{5}{9}$, $\frac{6}{7}$ and $\frac{2}{3}$.
24. $\frac{7}{8}$, $\frac{11}{16}$ and $\frac{17}{24}$.
25. $\frac{3}{2}$, $\frac{3}{4}$ and $\frac{7}{9}$.
26. $\frac{5}{7}$, $\frac{3}{14}$ and $\frac{10}{21}$.
27. $\frac{12}{20}$, $\frac{7}{15}$ and $\frac{2}{3}$.
28. $\frac{4}{13}$, $\frac{15}{26}$ and $\frac{7}{39}$.
29. $\frac{2}{3}$, $\frac{4}{13}$, $\frac{25}{26}$ and $\frac{4}{39}$
30. $\frac{6}{5}$, $\frac{7}{15}$, $\frac{5}{24}$ and $\frac{37}{60}$
31. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{13}{16}$ and $\frac{29}{96}$.
32. $\frac{11}{12}$, $\frac{7}{8}$, $\frac{11}{16}$ and $\frac{17}{24}$.

ADDITION.

240. ***Addition of Fractions*** is the process of finding the sum of two or more numbers, one or more of which are fractional.

241. PRINCIPLE.—*Fractions can be added only when they have a common denominator, and when they express fractional units of the same kind.*

WRITTEN EXERCISES.

1. Find the sum of $\frac{5}{6}$ and $\frac{3}{8}$.

OPERATION.

$$\tfrac{5}{6} + \tfrac{3}{8} = \tfrac{20}{24} + \tfrac{9}{24} = \tfrac{29}{24} = 1\tfrac{5}{24}$$

ANALYSIS.—Since the given fractions have not a common denominator, reduce them to equivalent fractions having 24 for their least common denominator. Then add their numerators and write the sum over the common denominator. Hence $\frac{5}{6} + \frac{3}{8} = 1\frac{5}{24}$.

2. Find the sum of $\frac{2}{5}$, $\frac{5}{6}$ and $\frac{3}{10}$; of $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{9}$.

3. What is the sum of $6\frac{1}{2}$, $7\frac{3}{4}$ and $3\frac{5}{6}$?

OPERATION.

$$\begin{aligned} 6\tfrac{1}{2} &= 6\tfrac{6}{12} \\ 7\tfrac{3}{4} &= 7\tfrac{9}{12} \\ 3\tfrac{5}{6} &= 3\tfrac{10}{12} \\ \hline 16 + \tfrac{25}{12} &= 18\tfrac{1}{12} \end{aligned}$$

ANALYSIS.—The sum of the fractions is $\frac{25}{12} = 2\frac{1}{12}$, which added to the sum of the integers 16, gives $18\frac{1}{12}$, the required result.

4. Find the sum of $8\frac{3}{4}$, $22\frac{4}{5}$ and 7; of 25, $41\frac{1}{8}$ and $\frac{5}{6}$; of $11\frac{7}{8}$, $16\frac{2}{3}$ and $4\frac{1}{2}$.

RULE.—I. *Reduce the given fractions to equivalent fractions having the least common denominator, then add their numerators, and write the sum over the common denominator.*

II. *When there are mixed numbers or integers, add the fractions and integers separately, and then add the results.*

1. If the mixed numbers are small, they may be reduced to improper fractions, and then added after the usual method.

2. The solution of an example in fractions should not be considered *complete* as long as the *result* contains an improper fraction, or one that can be reduced to lower terms.

Find the sum of

5. $\frac{3}{8}$, $\frac{3}{4}$ and $\frac{9}{10}$.
6. $\frac{6}{7}$, $\frac{2}{3}$ and $\frac{17}{21}$.
7. $\frac{5}{7}$, $\frac{11}{14}$ and $\frac{31}{42}$.
8. $\frac{4}{5}$, $\frac{9}{20}$ and $1\frac{5}{8}$.
9. $1\frac{1}{2}$, 2, $\frac{2}{3}$ and $3\frac{3}{4}$.
10. $\frac{7}{8}$, $15\frac{1}{4}$, $4\frac{1}{2}$ and $25\frac{1}{3}$.
11. $\frac{3}{4}$, $\frac{7}{16}$, $\frac{5}{6}$ and $4\frac{5}{12}$.
12. $30\frac{4}{5}$, $4\frac{9}{10}$, $10\frac{5}{6}$ and $57\frac{1}{3}$.
13. What is the sum of \$$14\frac{1}{5}$, \$$39\frac{7}{10}$, \$$16\frac{3}{8}$ and \$$25\frac{1}{4}$?

SUBTRACTION.

242. ***Subtraction of Fractions*** is the process of finding the difference of two numbers, one or both of which are fractional.

243. PRINCIPLE.—*One fraction can be subtracted from another, only when both have a common denominator, and when they express fractional units of the same kind.*

WRITTEN EXERCISES.

1. Subtract $\frac{3}{7}$ from $\frac{2}{3}$.

OPERATION.

$$\frac{2}{3} - \frac{3}{7} = \frac{14}{21} - \frac{9}{21} = \frac{5}{21}.$$

ANALYSIS.—Since the given fractions have not a common denominator, reduce them to equivalent fractions having 21 for their least common denominator. Then subtract the less numerator from the greater and write the difference over the common denominator. Hence $\frac{2}{3}-\frac{3}{7}=\frac{5}{21}$, the required difference.

2. Subtract $\frac{3}{4}$ from $\frac{5}{6}$.
3. Subtract $\frac{5}{8}$ from $\frac{11}{12}$.
4. Subtract $\frac{3}{8}$ from $\frac{6}{7}$.
5. Subtract $\frac{5}{12}$ from $\frac{4}{5}$.

6. From $16\frac{1}{3}$ take $7\frac{3}{4}$.

OPERATION.

$$\begin{array}{rcrcr} 16\frac{1}{3} & = & 16\frac{4}{12} & = & 15\frac{16}{12} \\ 7\frac{3}{4} & = & 7\frac{9}{12} & = & \underline{7\frac{9}{12}} \\ & & & & 8\frac{7}{12} \end{array}$$

ANALYSIS.—First reduce the fractional parts of the given numbers to their least common denominator 12. Since $\frac{9}{12}$ cannot be subtracted from $\frac{4}{12}$, take 1 or $\frac{12}{12}$ from 16, leaving 15, and add it to the $\frac{4}{12}$, making $\frac{16}{12}$. Then subtracting the fractions and integers separately we have $15\frac{16}{12}-7\frac{9}{12}=8\frac{7}{12}$, the required result.

RULE.—I. *Reduce the fractions when necessary to equivalent fractions having the least common denominator, and write the difference between the numerators over the common denominator.*

II. *When there are mixed numbers or integers, subtract the fractional and integral parts separately, and unite the results.*

If the mixed numbers are small, they may be reduced to improper fractions, and the subtraction performed according to the usual method

Find the difference between

7. $\frac{7}{8}$ and $\frac{7}{9}$.
8. $\frac{7}{20}$ and $\frac{5}{8}$.
9. $\frac{7}{12}$ and $\frac{21}{60}$.
10. $\frac{9}{14}$ and $\frac{16}{63}$.
11. $\frac{1}{15}$ and $\frac{5}{21}$.

12. $\frac{5}{7}$ and $\frac{4}{11}$.
13. $18\frac{1}{3}$ and $\frac{15}{16}$.
14. $27\frac{7}{8}$ and $16\frac{5}{9}$.
15. $48\frac{7}{25}$ and 32.
16. $126\frac{1}{2}$ and $75\frac{6}{7}$.

17. 97 and $13\frac{7}{20}$.
18. $327\frac{4}{11}$ and 450.
19. $106\frac{9}{38}$ and $214\frac{7}{11}$.
20. 200 and $306\frac{5}{17}$.
21. $421\frac{1}{9}$ and $245\frac{1}{6}$.

22. What number added to $12\frac{6}{7}$ will make $18\frac{1}{4}$?

23. A grocer sold tea at \$$\frac{15}{16}$ a pound that cost him \$$\frac{14}{20}$. What was his gain ?

24. From a cask of wine containing $60\frac{1}{2}$ gallons, $21\frac{5}{8}$ gallons were drawn. How much remained ?

ADDITION AND SUBTRACTION.

ORAL EXAMPLES.

244. *1.* How much less than 1 is $\frac{1}{3}+\frac{1}{5}$? $\frac{1}{4}+\frac{2}{5}$? $\frac{1}{2}+\frac{3}{7}$?

2. How much greater than 1 is $\frac{5}{6}+\frac{1}{2}$? $\frac{3}{8}+\frac{2}{3}$? $\frac{4}{5}+\frac{3}{4}$?

3. What is the difference between 3 and $1\frac{1}{2}$? 4 and $2\frac{4}{5}$?

4. Henry paid $\frac{1}{4}$ of all his money for a knife, $\frac{1}{8}$ for a ball, and $\frac{1}{6}$ for a necktie: what part of his money had he left?

5. A seedsman bought $2\frac{1}{2}$ bushels of clover seed, and then sold $\frac{3}{8}$ of a bushel to one man, and $\frac{3}{4}$ to another: how much had he left?

6. A real estate broker sold from a piece of land at different times $\frac{1}{3}$, $\frac{3}{8}$ and $\frac{1}{4}$ of it: how much remained unsold?

7. What is the difference between $\frac{1}{2}+\frac{4}{5}$ and $\frac{1}{4}+\frac{7}{10}$?

8. How much greater than $2\frac{7}{8}$ is $1\frac{5}{6}+3\frac{1}{4}$?

9. From the sum of $\frac{3}{5}$ and $\frac{5}{6}$ take their difference.

10. Paid \$$6\frac{3}{8}$ for a ton of coal, and \$$2\frac{1}{6}$ for a load of wood: how much change must be returned for \$10?

11. How do you subtract fractions having different denominators one from the other?

12. Subtract $\frac{1}{2}$ from $\frac{3}{4}$; $\frac{2}{3}$ from $\frac{5}{6}$; $\frac{3}{4}$ from $\frac{7}{8}$; $\frac{2}{5}$ from $\frac{5}{6}$.

13. If a grocer buys tea at \$$\frac{5}{8}$ a pound and sells it at \$$\frac{3}{4}$, does he gain or lose, and how much?

Find the second member of the following equations:

14. $\frac{3}{4}+\frac{2}{3}-\frac{1}{2}=?$
15. $\frac{7}{8}-\frac{1}{6}+\frac{1}{4}=?$
16. $\frac{9}{10}+2-1\frac{1}{2}=?$
17. $4\frac{1}{7}+\frac{9}{14}-3\frac{2}{7}=?$
18. $12-\frac{7}{12}+7\frac{1}{4}=?$
19. $(\frac{5}{6}+\frac{2}{3})-(\frac{1}{2}+\frac{1}{4})=?$
20. $2\frac{1}{5}-\frac{3}{10}+4\frac{3}{4}=?$
21. $8\frac{4}{9}-(6\frac{2}{3}-\frac{2}{9})=?$
22. $\frac{9}{16}+5\frac{1}{2}+3\frac{1}{4}=?$
23. $12\frac{13}{20}+5\frac{4}{5}-2\frac{1}{2}=?$

WRITTEN EXERCISES.

245. *1.* From the sum of $\frac{6}{7}$, $\frac{5}{8}$ and $\frac{3}{4}$ take $\frac{13}{14}$.

2. From the sum of $\frac{9}{10}$ and $\frac{7}{8}$ take the difference of $1\frac{1}{4}$ and $\frac{5}{16}$.

3. A man owning $\frac{19}{27}$ of a cotton mill, sold $\frac{1}{3}$ of the whole to one man and $\frac{2}{9}$ to another: what part had he left?

4. To what must $\frac{2}{5}$ be added that the sum may be $\frac{3}{4}$?

5. What number must be added to the sum of $1\frac{7}{20}$ and $2\frac{2}{3}$ to make $36\frac{1}{4}$?

6. From the sum of $14\frac{1}{2}$ and $9\frac{1}{3}$ take the difference of $25\frac{3}{4}$ and $16\frac{1}{3}$.

7. From the sum of $\frac{7}{9}$ and $\frac{17}{21}$ take the difference of $\frac{4}{7}$ and $\frac{2}{3}$.

8. The sum of two numbers is $14\frac{7}{24}$, and the less is $3\frac{1}{8}$: what is the greater?

9. The sum of two fractions is $\frac{37}{48}$, and the greater is $\frac{5}{8}$: what is the less?

10. A clerk earned $\$75\frac{3}{4}$ one month, and paid for board $\$28\frac{1}{3}$, for washing $\$7\frac{1}{2}$, and other expenses $\$27\frac{7}{16}$: how much had he left?

11. A man bequeathed $\frac{5}{18}$ of his property to his wife, $\frac{7}{12}$ to his son, and the remainder to his daughter: what part did the daughter receive?

12. The difference of two numbers is $7\frac{1}{3}$, and the less is $20\frac{3}{4}$: what is the greater?

13. The sum of two numbers is $26\frac{15}{32}$, and the greater is $15\frac{5}{8}$: what is the less?

14. A lady having $\$50$, paid $\$12\frac{3}{8}$ for a bonnet, $\$17\frac{5}{8}$ for a shawl, and $\$3\frac{7}{10}$ for a veil: how much money had she left?

MULTIPLICATION.

246. ***Multiplication of Fractions*** is the process of finding the product of two or more factors, one or more of which are fractions. (Review pages 61 and 62.)

ORAL EXERCISES.

1. At $\$\frac{3}{8}$ a pound, what will 4 pounds of sugar cost?

ANALYSIS.—Four pounds will cost 4 times $\$\frac{3}{8}$, or $\$\frac{12}{8}$, equal to $\$1\frac{1}{2}$. Hence 4 pounds of sugar will cost $\$1\frac{1}{2}$.

2. How much is 4 times $\frac{3}{4}$ of a mile? 7 times $\frac{5}{8}$ of a yard? 10 times $\frac{4}{7}$ of a week?

3. At $\$6\frac{2}{3}$ a ton, what will 5 tons of coal cost?

ANALYSIS.—Five tons of coal will cost 5 times $\$6\frac{2}{3}$; 5 times $\$\frac{2}{3}$ are $\$\frac{10}{3}$, equal to $\$3\frac{1}{3}$; 5 times \$6 are \$30, and $\$3\frac{1}{3}$ added make $\$33\frac{1}{3}$.

4. What cost 7 oranges at $4\frac{1}{2}$ cents each?

5. How much is 4 times $5\frac{3}{4}$? 6 times $7\frac{3}{8}$? 5 times $12\frac{4}{7}$?

6. Multiplying by $\frac{1}{4}$ is the same as dividing by what?

When a *fractional part* of an integer or fraction is to be taken, the word "*of*," and not "*times*," should be used.

7. What is $\frac{1}{6}$ of 5 miles?

ANALYSIS.—$\frac{1}{6}$ of 5 miles is 5 times $\frac{1}{6}$ of 1 mile, or $\frac{5}{6}$ of 1 mile.

8. What is $\frac{1}{8}$ of 16? $\frac{3}{4}$ of 16? $\frac{3}{8}$ of 24? $\frac{5}{6}$ of 30?

9. What is $\frac{3}{7}$ of 25? $\frac{4}{5}$ of 36? $\frac{5}{6}$ of 44? $\frac{4}{9}$ of 65?

10. At \$5 a yard, what will $\frac{3}{8}$ of a yard of cloth cost?

ANALYSIS.—Since 1 yard cost \$5, $\frac{3}{8}$ of a yard will cost $\frac{3}{8}$ of \$5, or 3 times $\frac{1}{8}$ of \$5. $\frac{1}{8}$ of \$5 is $\$\frac{5}{8}$, and 3 times $\$\frac{5}{8}$ are $\$\frac{15}{8}$, or $\$1\frac{7}{8}$.

11. At \$7 a gross, what will $\frac{7}{8}$ of a gross of bottles cost?

12. At \$8 a ton, what will $\frac{2}{3}$ of a ton of coal cost?

13. At $\$7$ a ton, what will $4\frac{3}{4}$ tons of coal cost?

Analysis.—Since 1 ton cost $\$7$, $4\frac{3}{4}$ tons will cost $4\frac{3}{4}$ times $\$7$. 4 times $\$7$ are $\$28$, and $\frac{3}{4}$ of $\$7$ is $\$5\frac{1}{4}$, which added to $\$28$, make $\$33\frac{1}{4}$.

14. If a man earn $\$4$ a day, how much will he earn in $6\frac{2}{3}$ days? In $5\frac{1}{6}$ days? In $8\frac{3}{4}$ days? In $12\frac{1}{2}$ days?

15. If a barrel of flour last a family 6 weeks, how long will $5\frac{3}{4}$ barrels last them? $4\frac{3}{8}$ barrels? $7\frac{4}{5}$ barrels?

16. If a pound of coffee costs $\$\frac{3}{8}$, what cost $\frac{2}{3}$ of a pound?

Analysis.—The cost will be $\frac{2}{3}$ of $\$\frac{3}{8}$, or 2 times $\frac{1}{3}$ of $\$\frac{3}{8}$. $\frac{1}{3}$ of $\$\frac{3}{8}$ is $\frac{3}{24}$ or $\$\frac{1}{8}$, and 2 times $\$\frac{1}{8}$ is $\frac{2}{8}$ or $\$\frac{1}{4}$. Hence, etc.

Fractions with the word *of* between them are sometimes called *Compound Fractions.* The word *of* is equivalent to the sign (×) of multiplication. Thus, $\frac{5}{9}$ *of* $\frac{2}{3} = \frac{5}{9} \times \frac{2}{3}$; $\frac{4}{5}$ *of* $9 = \frac{4}{5} \times 9$, etc.

17. What is $\frac{2}{3}$ of $\frac{6}{7}$? $\frac{4}{5}$ of $\frac{3}{4}$? $\frac{5}{6}$ of $\frac{3}{8}$? $\frac{3}{7}$ of $\frac{5}{2}$?

18. At $\$\frac{4}{5}$ a gallon, what will $\frac{3}{4}$ of a gallon of syrup cost?

WRITTEN EXERCISES.

1. Multiply $\frac{15}{16}$ by $\frac{8}{9}$.

OPERATION.

$$\frac{15}{16} \times \frac{8}{9} = \frac{120}{144} = \frac{5}{6}$$

$$\text{Or, } \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{\underset{3}{\cancel{9}}} = \frac{5}{6}$$

Analysis.—To multiply $\frac{15}{16}$ by $\frac{8}{9}$ is to find $\frac{8}{9}$ of $\frac{15}{16}$, which is 8 times $\frac{1}{9}$ of $\frac{15}{16}$; $\frac{1}{9}$ of $\frac{15}{16}$ is $\frac{15}{16 \times 9}$; and $\frac{8}{9}$ of $\frac{15}{16}$ is 8 times $\frac{15}{16 \times 9}$, or $\frac{15 \times 8}{16 \times 9} = \frac{120}{144} = \frac{5}{6}$, the required product. Or, shortening the operation by cancelling equal factors in the numerators and denominators, we obtain $\frac{5}{6}$, the same result.

Rule.—I. *Reduce all integers and mixed numbers to improper fractions.*

II. *Find the product of the numerators for a new numerator, and of the denominators for a new denominator.*

Use cancellation when convenient, both to *shorten* the operation, and to give the answer in its *lowest terms.*

2. When peaches are worth $\$\frac{7}{8}$ a basket, what will $\frac{3}{4}$ of $\frac{4}{7}$ of a basket cost?

3. At $\$\frac{5}{8}$ a gallon, what will be the cost of $\frac{4}{5}$ of 3 gallons of molasses?

What is the product

4. Of $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{4}{5}$?
5. Of $\frac{2}{9}$, $\frac{1}{2}$ and $\frac{5}{8}$?
6. Of $\frac{2}{7}$, $\frac{7}{9}$ and $\frac{5}{10}$?
7. Of $\frac{5}{6}$, $\frac{8}{15}$ and $\frac{3}{7}$?
8. Of $2\frac{3}{7}$, $\frac{5}{11}$ and $\frac{22}{25}$?
9. Of $5\frac{1}{2}$, $\frac{7}{11}$ and $\frac{6}{7}$?
10. Of $12\frac{3}{5}$, $\frac{2}{6}$ and 7?
11. Of $\frac{3}{4}$, $\frac{2}{5}$, $1\frac{2}{3}$ and $\frac{10}{11}$?

Find the value of

12. $\frac{1}{3}$ of $\frac{2}{5}$ of $2\frac{1}{2}$.
13. $\frac{5}{9}$ of $\frac{3}{11}$ of 4.
14. $\frac{2}{7}$ of $\frac{8}{13}$ of $3\frac{1}{4}$.
15. $\frac{3}{4}$ of $\frac{6}{7} \times \frac{4}{9}$ of $\frac{1}{6}$.
16. $\frac{4}{5}$ of $3 \times \frac{7}{8}$ of $2\frac{1}{2}$.
17. $\frac{3}{5}$ of $\frac{1}{6} \times \frac{2}{3}$ of 10.
18. $\frac{5}{12}$ of $4 \times \frac{1}{4}$ of $3\frac{3}{7}$.
19. $\frac{9}{10} \times 2\frac{1}{2}$ of $\frac{5}{9} \times 1\frac{1}{3}$.
20. $\frac{6}{7}$ of $\frac{7}{11} \times 5\frac{1}{2}$ times $\frac{5}{6}$.
21. $2\frac{1}{7}$ times $14 \times 1\frac{1}{3}$ times $2\frac{1}{4}$.

Find the cost of the following:

22. $\frac{7}{9}$ of a yard of flannel, at $\$\frac{3}{8}$ a yard.
23. $6\frac{1}{4}$ pounds of rice at $6\frac{1}{2}$ cents a pound.
24. $8\frac{3}{4}$ bushels of apples at $37\frac{1}{2}$ cents a bushel.
25. $21\frac{4}{5}$ yards of calico at $\$\frac{1}{8}$ a yard.
26. 15 tons of coal at $\$7\frac{5}{8}$ a ton.
27. $9\frac{3}{5}$ pounds of coffee, at $28\frac{1}{2}$ cents a pound.
28. 56 pounds of cotton, at $18\frac{3}{4}$ cents a pound.
29. $\frac{2}{3}$ of $5\frac{1}{2}$ pounds of wool, at 47 cents a pound.
30. $\frac{4}{5}$ of 9 pounds of honey, at $\frac{1}{2}$ of $18\frac{3}{4}$ cents a pound.
31. $5\frac{1}{4}$ quarts of chestnuts, at $12\frac{1}{2}$ cents a quart.

32. If an acre of land be worth \$240, what is $\frac{7}{15}$ of it worth? $\frac{5}{8}$ of it? $\frac{11}{20}$ of it? $\frac{9}{16}$ of it?

33. If a man travel 36 miles a day, how far can he travel in $16\frac{5}{8}$ days? In $12\frac{7}{8}$ days?

DIVISION.

247. ***Division of Fractions*** is the process of finding the quotient when either the dividend or divisor or both contain fractions. (Review page 63.)

ORAL EXERCISES.

1. If $\frac{2}{3}$ of a dollar be equally divided between 2 boys, what part of a dollar will each have?

2. What is the quotient of $\frac{2}{3}$ divided by 2?

3. $\frac{1}{2}$ of $\frac{2}{3}$ is what part of 1?

4. Dividing by 2 is the same as multiplying by what fraction?

5. Dividing by 3 and 4 is the same as multiplying by what fractions? $\frac{1}{3}$ of $\frac{3}{4}$ is what part of 1?

6. If 3 yards of tape cost \$$\frac{2}{3}$, what will 1 yard cost?

ANALYSIS.—1 yard will cost $\frac{1}{3}$ of \$$\frac{2}{3}$, or \$$\frac{2}{9}$. Hence, etc.

7. Divide $\frac{8}{15}$ by 4; $\frac{10}{21}$ by 5; $\frac{3}{11}$ by 7; $\frac{5}{6}$ by 9.

8. If 6 pounds of sugar cost \$$\frac{3}{5}$, what will 1 pound cost?

9. If 1 man can do $\frac{5}{8}$ of a piece of work in 1 day, in what time can 4 men do it? 3 men? 5 men?

10. If 5 men can build $\frac{6}{7}$ of a wall in a day, what part of it can 1 man build? 2 men? 3 men? 4 men?

11. If 3 lemons cost $\frac{4}{5}$ of a dime, what cost 1 lemon?

12. If 4 slates cost \$$\frac{3}{9}$, what will 1 slate cost?

13. John sold $\frac{2}{3}$ of 6 quarts of chestnuts for \$$\frac{2}{5}$, what was that a quart?

14. Divide $\frac{5}{8}$ by $\frac{3}{5}$ of 20; $\frac{12}{17}$ by $\frac{3}{5}$ of 15; $\frac{7}{9}$ by $\frac{1}{8}$ of 40.

15. What is the quotient of 1 divided by $\frac{1}{2}$? By $\frac{1}{3}$? by $\frac{1}{5}$? By $\frac{1}{6}$? By $\frac{1}{8}$? By $\frac{1}{9}$? By $\frac{1}{12}$?

16. When potatoes are $\$\frac{2}{3}$ a bushel, how many bushels can be bought for \$5?

ANALYSIS.—As many bushels as $\$\frac{2}{3}$ is contained times in \$5; \$5 is equal to $\$\frac{15}{3}$, and 2 thirds is contained in 15 thirds $7\frac{1}{2}$ times. Hence $7\frac{1}{2}$ bushels can be bought for \$5.

17. 2 are how many times $\frac{1}{8}$? $\frac{3}{8}$? $\frac{2}{5}$? $\frac{4}{5}$? $\frac{2}{7}$? $\frac{5}{7}$? $\frac{5}{8}$? $\frac{4}{9}$?

18. If a horse eat $\frac{3}{7}$ of a bushel of oats in a day, how long will 4 bushels last him? 5 bushels? 6 bushels?

19. In how many days can a boy earn \$6, if he earn $\$\frac{3}{4}$ a day? If $\$\frac{5}{6}$? If $\$\frac{7}{8}$.

20. How many times is $\frac{2}{7}$ contained in $\frac{6}{7}$? $\frac{3}{8}$ in $\frac{6}{8}$? $\frac{4}{9}$ in $\frac{8}{9}$?

21. At $\$\frac{2}{5}$ a yard, how many yards of ribbon can be bought for $\$\frac{3}{4}$?

ANALYSIS.—As many yards as $\$\frac{2}{5}$ is contained times in $\$\frac{3}{4}$; $\$\frac{2}{5}$ equals $\$\frac{8}{20}$, and $\$\frac{3}{4}$ equals $\$\frac{15}{20}$; 8 twentieths is contained in 15 twentieths $1\frac{7}{8}$. Hence $1\frac{7}{8}$ yards can be bought for $\$\frac{3}{4}$.

22. Divide $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{6}{7}$ by $\frac{1}{2}$; $\frac{5}{7}$ by $\frac{1}{3}$; $\frac{7}{8}$ by $\frac{3}{4}$? $\frac{7}{12}$ by $\frac{1}{4}$?

23. How do you divide one fraction by another when they have not a common denominator?

24. At $\$\frac{3}{16}$ a yard, how many yards of muslin can be bought for $\$\frac{1}{2}$? For $\$\frac{3}{4}$? For $\$\frac{7}{8}$?

25. If $\frac{2}{7}$ of a drum of figs cost $\$\frac{5}{8}$, what will 1 drum cost?

ANALYSIS.—Since $\frac{2}{7}$ of a drum of figs cost $\$\frac{5}{8}$, $\frac{1}{7}$ of a drum will cost $\frac{1}{2}$ of $\$\frac{5}{8}$, which is $\$\frac{5}{16}$; and 1 drum, or $\frac{7}{7}$, will cost 7 times $\$\frac{5}{16}$, or $\$\frac{35}{16}$, equal to $\$2\frac{3}{16}$.

26. If $\frac{2}{3}$ of a yard of silk cost $\$\frac{3}{4}$, what will 1 yard cost?

27. At $\$\frac{7}{8}$ a yard, how many yards of silk can be bought for $\$1\frac{5}{8}$? For $\$2\frac{1}{4}$? For $\$3\frac{1}{8}$?

WRITTEN EXERCISES.

248. *1.* Divide $\frac{7}{8}$ by $\frac{2}{3}$.

OPERATION.

$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16} = 1\frac{5}{16}$.

ANALYSIS. — Since $\frac{2}{3}$ is contained in 1, $\frac{3}{2}$ times, in $\frac{7}{8}$ it is contained $\frac{7}{8}$ of $\frac{3}{2}$ times, or $\frac{21}{16}$, equal to $1\frac{5}{16}$.

It will be observed that the operation is equivalent to multiplying the dividend $\frac{7}{8}$ by the divisor $\frac{2}{3}$ with its terms *inverted*.

2. Divide $3\frac{1}{2}$ by $4\frac{3}{8}$.

OPERATION.

$3\frac{1}{2} = \frac{7}{2}$; $4\frac{3}{8} = \frac{35}{8}$

$\frac{7}{2} \div \frac{35}{8} = \frac{\cancel{7}}{\cancel{2}} \times \frac{\cancel{8}^{4}}{\cancel{35}_{5}} = \frac{4}{5}$

ANALYSIS.—Reduce both dividend and divisor to improper fractions. Then divide as in the first example.

RULE. I.—*Reduce all integers and mixed numbers to improper fractions.*

II.—*Multiply the dividend by the terms of the divisor, inverted.*

Use *cancellation* when convenient.

Divide

3. $\frac{36}{43}$ by 18.
4. $\frac{27}{12}$ by 10.
5. $\frac{2}{70}$ by 14.
6. $\frac{44}{97}$ by 21.

7. $73\frac{3}{4}$ by 10.
8. $125\frac{4}{7}$ by 20.
9. $236\frac{1}{8}$ by 16.
10. $328\frac{2}{5}$ by 18.

11. $\frac{1}{4}$ of 30 by 12.
12. $\frac{1}{4}$ of 142 by 15.
13. $\frac{5}{6}$ of 8 by 20.
14. $\frac{5}{10}$ of 75 by 11.

15. If a man dig $29\frac{5}{16}$ rods of ditch in 14 days, how much does he dig in 1 day?

16. If 8 tons of coal cost \$$47\frac{7}{8}$, what is the cost of 1 ton?

17. If $\frac{3}{4}$ of 12 yards of silk cost \$$39\frac{3}{8}$, what is the cost of 1 yard?

18. If 12 plows cost \$$124\frac{3}{4}$, what is the cost of each?

19. What number multiplied by 48 produces $694\frac{3}{8}$?

20. If 54 horses cost \$$4622\frac{2}{5}$, what is the cost of each?

Divide

21. 56 by $\frac{6}{11}$.
22. 84 by $\frac{7}{9}$.
23. 163 by $\frac{7}{13}$.
24. 120 by $\frac{6}{11}$.
25. 72 by $\frac{11}{18}$.
26. 90 by $\frac{15}{21}$.
27. 300 by $\frac{7}{8}$.
28. 155 by $\frac{9}{25}$.
29. 168 by $3\frac{4}{7}$.
30. 240 by $\frac{5}{6}$ of $1\frac{1}{2}$.
31. 189 by $\frac{3}{4}$ of $\frac{4}{5}$.
32. 106 by $6\frac{4}{9}$.

33. If $27 be divided among some laborers, giving them $$\frac{9}{10}$ each, how many laborers will receive a share?

34. At $$3\frac{3}{8}$ a cord, how many cords of wood can be bought for $40? For $150?

35. How many bushels of pears at $$\frac{5}{8}$ a bushel can be bought for $18? For $39?

36. At $$6\frac{2}{3}$ a ton, how many tons of coal can be bought for $160? For $248?

37. At $$\frac{11}{12}$ a yard, how many yards of cloth can be bought for $9? For $24? For $64?

Divide

38. $\frac{7}{36}$ by $\frac{14}{27}$.
39. $\frac{20}{21}$ by $\frac{1}{2}$ of $\frac{4}{5}$.
40. $\frac{24}{63}$ by $\frac{2}{3}$ of $1\frac{5}{9}$.
41. $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{7}{8}$.
42. $\frac{11}{15}$ by $\frac{1}{7}$ of $\frac{3}{5}$.
43. $9\frac{1}{6}$ by $\frac{1}{2}$ of 5.
44. $4\frac{5}{7}$ by $2\frac{3}{4}$.
45. $12\frac{9}{10}$ by $\frac{1}{3}$ of $6\frac{3}{4}$.

46. If $\frac{3}{5}$ of a farm cost $6270, what did the whole cost?

47. How many pounds of tea at $$1\frac{1}{8}$ a pound, can be bought for $$15\frac{1}{3}$?

48. At $$3\frac{1}{5}$ a cord, how many cords of wood can be bought for $25?

49. If a man spend $$\frac{3}{10}$ a day for cigars, in what time will he spend $$5\frac{3}{8}$?

50. If $1\frac{1}{3}$ bushels of oats cost $$\frac{5}{8}$, what costs 1 bushel?

51. How many barrels of flour at $9 a barrel, can be bought for $\frac{7}{12}$ of $270?

REVIEW.

ORAL EXERCISES.

249. *1.* How many sixths in 4? In $5\frac{5}{6}$? In $8\frac{1}{6}$?

2. Express $\frac{3}{4}$ in higher terms? $\frac{5}{6}$; $\frac{4}{5}$; $\frac{7}{8}$; $\frac{9}{10}$.

3. Express $\frac{9}{12}$ in lower terms; $\frac{16}{20}$; $\frac{24}{30}$; $\frac{20}{40}$.

4. Reduce $\frac{8}{12}$ and $\frac{12}{20}$ to their lowest terms.

5 Change $\frac{3}{5}$ and $\frac{1}{4}$ to twentieths; $\frac{1}{2}$ and $\frac{1}{3}$ to twelfths

6. Which is greater, $\frac{2}{7}$ or $\frac{1}{3}$? $\frac{1}{4}$ or $\frac{3}{10}$? $\frac{2}{3}$ or $\frac{5}{7}$?

7. Name 3 equivalent fractions for $\frac{1}{5}$; for $\frac{2}{3}$; for $\frac{3}{4}$.

8. Change $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ to fractions having 48 for a common denominator.

9. Find the least common denominator of the same.

10. For $\frac{16}{24}$ name 3 equivalent fractions in lower terms?

11. Change $\frac{2}{3}$, $1\frac{1}{2}$ and 2 to sixths; $\frac{4}{5}$ and $2\frac{1}{2}$ to twentieths.

12. A farmer sold $\frac{1}{3}$ of his grain to one man; $\frac{2}{5}$ to another, and $\frac{1}{6}$ to another; what part of his grain did he sell, and what part had he left?

13. A lady paid $\frac{1}{3}$ of her money for a bonnet, $\frac{1}{4}$ for a shawl, and $\frac{1}{5}$ for a pair of gaiters. What had she left?

14. From 4 tons of coal $\frac{4}{5}$ of a ton was used one month and $1\frac{1}{3}$ tons the next; how much was left?

15. If $\frac{1}{4}$ of a box of raisins cost $\$2\frac{1}{12}$, what will $\frac{5}{6}$ of a box cost?

16. If $\frac{3}{5}$ of a yard of silk cost \$3, how many yards can be bought for \$15?

17. Henry bought $\frac{1}{2}$ a box of figs, and John $\frac{1}{2}$ as many. What part of a whole box did both buy?

18. Bought a watch and chain. The chain cost \$30, which was $\frac{3}{5}$ of what the watch cost. What did both cost?

WRITTEN EXERCISES.

250. *1.* Reduce to their lowest terms $\frac{576}{864}$; $\frac{9504}{10368}$.

2. Change to integers or mixed numbers, $\frac{5907}{25}$; $\frac{85641}{39}$.

3. Reduce to improper fractions $164\frac{5}{14}$; $47\frac{42}{61}$; $104\frac{9}{112}$.

4. Change 21 to a fraction having 25 for its denominator; 34 to 50ths; 75 to 18ths.

5. Reduce $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{3}{5}$, $1\frac{5}{8}$ to equivalent fractions having the least common denominator

6. What is the difference between $\frac{3}{4}$ of $6\frac{1}{3}$ and $4\frac{1}{2} + \frac{5}{6}$?

7. A farmer had $\frac{1}{4}$ of his wheat in 1 bin, $\frac{1}{3}$ in another, and 60 bushels in the third. How many bushels had he?

8. Paid \$$1\frac{1}{3}$ a bushel for 10 bushels of wheat, and \$$\frac{5}{6}$ a bushel for 14 bushels of corn; which cost the more?

9. How many yards of cloth $\frac{4}{5}$ of a yard wide, are equal to 12 yards $\frac{3}{4}$ of a yard wide?

10. How many pounds of tea at \$$\frac{7}{8}$ a pound will pay for $10\frac{1}{2}$ baskets of peaches, at \$$\frac{9}{10}$ a basket?

11. If $\frac{3}{10}$ of an acre of land cost \$$79\frac{1}{5}$, what cost 1 acre?

12. At \$$\frac{3}{8}$ a pound, how many pounds of coffee can be bought for \$$6\frac{1}{2}$?

13. If $7\frac{1}{2}$ barrels of pork cost \$60, how many barrels can be bought for \$156 at the same rate?

14. A pole stands $\frac{1}{6}$ in the mud, $\frac{1}{4}$ in the water, and 35 feet above the water. What is the length of the pole?

15. A certain sum of money is divided among 4 persons. A has $\frac{2}{7}$, B, $\frac{1}{4}$, C, $\frac{3}{14}$, and D has the remainder, which is \$30. What was the whole sum divided?

16. If 3 yards of velvet cost \$$7\frac{3}{4}$, how much more than \$$10\frac{1}{2}$ will 5 yards cost?

DRILL EXERCISES.

251. *1.* Practice pupils in *halving* and *doubling* fractions. Take $\frac{1}{2}$. Teacher dictates, "Half of $\frac{1}{2}$;" "Half of $\frac{1}{4}$;" "Half of $\frac{1}{8}$," the pupils naming the successive results to $\frac{1}{8192}$. (See **Art. 28.**)

2. Reverse the exercise; thus, "Twice $\frac{1}{8192}$;" "Twice $\frac{1}{4096}$," etc., the pupils naming results as before.

3. In like manner, dictate "One-fifth of $\frac{1}{5}$," etc., and reverse.

4. Also, "One-tenth of $\frac{1}{10}$;" "One-tenth of $\frac{1}{100}$;" "One-tenth of $\frac{1}{1000}$;" and reverse the process, beginning, "Ten times $\frac{1}{10000}$;" "Ten times $\frac{1}{1000}$;" "Ten times $\frac{1}{100}$;" "Ten times $\frac{1}{10}$;" "Ten times 1;" "Ten times 10;" "Ten times 100;" "Ten times 1000," to "Ten times 100000."

5. Compare the scales of increase and decrease in the following examples:

8,	4,	2,	1,	$\frac{1}{2}$,	$\frac{1}{4}$,	$\frac{1}{8}$.
125,	25,	5,	1,	$\frac{1}{5}$,	$\frac{1}{25}$,	$\frac{1}{125}$.
1000,	100,	10,	1,	$\frac{1}{10}$,	$\frac{1}{100}$,	$\frac{1}{1000}$.

Observe, that only when the scale of increase is 10, the successive products at the left of 1 correspond with the successive orders in numeration of integers.

6. Begin at 8 and halve to $\frac{1}{8}$.

7. Begin at $\frac{1}{8}$ and double to 8.

8. In the same manner with the scales of 5 and 10.

When this page, together with Pages 94 and 125, are thoroughly mastered, the principles of decimal notation as applied to fractions in the following pages will be easily understood.

DECIMALS.

252. The numbers 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., increase from right to left and decrease from left to right, by the same uniform scale of *ten*. Thus, 100 is ten times 10; 10 is ten times 1; 1 is ten times $\frac{1}{10}$, etc. Also, $\frac{1}{100}$ is one-tenth of $\frac{1}{10}$; $\frac{1}{10}$ is one-tenth of 1; 1 is one-tenth of 10, etc. The value of any decimal figure is always $\frac{1}{10}$ of the value of the same figure in the next place to the left.

253. As the decimal *multiples* of a unit are expressed in orders at the *left* of the order of units, so the decimal *divisions* of a unit can be expressed in orders at the *right* of the order of units without their denominators.

These fractions, when so written, are called ***Decimals.***

254. At the left of the order of units, the orders are called *tens, hundreds, thousands,* etc. At the right, they are called *tenths, hundredths, thousandths,* etc. By extending the orders to the *left,* we may express any number however large, and by extending the orders to the *right,* we may express any decimal division of a number however small.

255. The unit is the standard by which we compare all numbers. Therefore, to determine the order of units at a glance, a point is placed on the right of the order of units, and before the order of tenths.

It is written thus, (.), and is called the ***decimal point.*** Its only purpose is to determine the place of the order of units, and it must never be omitted.

Thus, in the number 62.55, the *point* shows that 2 stands in the order of units, and therefore the whole number to the left of the point is *sixty-two.*

As we read the 6 tens and 2 units at the left of the point, as sixty-two *units*, giving to it the name of the lowest order, so we read the 5 tenths and 5 hundredths as fifty-five *hundredths*, giving to *it* also the name of the lowest order, for $\frac{5}{10}+\frac{5}{100}=\frac{55}{100}=.55$.

256. Decimal fractions may be written in two ways; either as *other fractions*, the denominator being expressed, or, in *decimal notation*, the denominator being omitted. Thus,

$\frac{5}{10}$, or .5 is read 5 tenths and is $\frac{1}{10}$ of 5 units.
$\frac{5}{100}$, " .05 " 5 hundredths, " $\frac{1}{10}$ " 5 tenths.
$\frac{5}{1000}$, " .005 " 5 thousandths, " $\frac{1}{10}$ " 5 hundredths.

257. The name of the first order at the right of units is *tenths ;* of the second, *hundredths ;* of the third, *thousandths ;* of the fourth, *ten-thousandths,* etc.

Hence to read Decimals,

RULE.—*Read them as whole numbers, giving to them the name of the right-hand order.*

258. When an integer and decimal are written together, the expression is a ***Mixed Number*** **(60)**.

Mixed numbers involving decimals are read in the same manner as mixed numbers involving common fractions.

Thus, .07 is read *seven hundredths*, .58 is read *fifty-eight hundredths*, 625.348 is read *six hundred twenty-five and three hundred forty-eight thousandths.*

259. The relation of decimals and integers is clearly shown in the following Table:

Ten-thousands.	Thousands.	Hundreds.	Tens.	UNITS.	.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.
		4	2	5	.	3	7		
	1	0	4	6	.	0	6	5	
3	2	4	0	0	.	4	5	6	
			8	7	.	0	0	7	5
	4	3	2	8	.	1	6	9	
	7	0	0	0	.	0	0	0	7

INTEGERS. DECIMALS.

Read as follows, always using the word *"and,"* between the integer and decimal:

425 and 37 *hundredths.*
1046 and 65 *thousandths.*
32400 and 456 *thousandths.*
87 and 75 *ten-thousandths.*
4328 and 169 *thousandths.*
7000 and 7 *ten-thousandths.*

260. Read the following numbers, naming all the orders, until the names of the decimal orders are as familiar as those of the integers:

1. .7.
2. .07.
3. .77.
4. .007.
5. .077.
6. .777.
7. .0007
8. .0077.
9. .0777.
10. .7777.

11. .6.
12. .67.
13. .008.
14. .327.
15. .6489.
16. .075.
17. .673.
18. .4859.
19. .0039.
20. .0476.

21. 5.5.
22. 50.05.
23. 55.55.
24. 500.005.
25. 550.005.
26. 555.555.
27. 5000.0005.
28. 5500.0055.
29. 5550.0555.
30. 5555.5555.

Additional examples may be given at the option of the teacher.

261. Decimals are governed by the same laws of notation as integers, and all the operations are performed precisely as the same operations in whole numbers.

ADDITION.

WRITTEN EXERCISES.

262. *1.* What is the sum of 2.125, 13.07, 7.8, and .3142?

OPERATION.

$$\begin{array}{r} 2.1250 \\ 13.0700 \\ 7.8000 \\ .3142 \\ \hline 23.3092 \end{array}$$

ANALYSIS.—Write the numbers so that units of the same order stand in the same column. After reducing the decimals to a common denominator by annexing ciphers, or supposing them to be annexed, add as in integers, placing the decimal point before *tenths* in the sum.

RULE—I. *Write the numbers so that units of the same order shall stand in the same column.*

II. *Add as in addition of integers, and place the decimal point before the order of tenths in the sum.*

(*2.*)	(*3.*)	(*4.*)	(*5.*)
42.3	12.326	4031.06	.6082
13.06	204.09	108.304	.0315
8.049	8.3024	9.0013	.8034
1.6	52.007	76.739	.086
.037	324.1	250.0007	.9106

In like manner find the sum

6. Of 405.327, 64.03, .84673, 121.8, and 7.00327.

7. Of $18.19, $142.095, $.964, $5.125, and $40.50.

8. A grocer bought a barrel of sugar for $21.84, a box of tea for $42.375, a cheese for $6.08, and a tub of butter for $10.125. What was the cost of all?

SUBTRACTION.

WRITTEN EXERCISES.

263. *1.* From 7.6325 subtract 5.4.

OPERATION.
7.6325
5.4000
2.2325

ANALYSIS.—Write the given numbers as in Addition, the subtrahend under the minuend, reducing the decimals to a common denominator, by annexing ciphers, or supposing them to be annexed, and then subtract as in integers.

RULE.—I. *Write the subtrahend under the minuend, so that units of the same order stand in the same column.*

II. *Subtract as in subtraction of integers, and place the decimal point before the orders of tenths in the remainder.*

	(2.)	*(3.)*	*(4.)*	*(5.)*
From	18.5	2.8706	.50376	.36
Subtract	2.3476	.49	.065	.127084

6. From $125.75 subtract $41.095.

7. From $1.375 subtract $.88.

8. From .84032 subtract .0047.

9. A man having $1000 in bank, drew out $450.62. How much remained in bank?

10. Sold a horse for $300, which was $62.25 more than he cost. What did he cost?

11. Bought a farm for $2560.50; paid at one time $1046, and at another, $807.87. What remained unpaid?

12. A man having $14725, gave $3560 for a store, and $7015.87 for goods. How much money had he left?

13. Bought groceries to the amount of $7.18. How much change must I receive for two five-dollar bills?

MULTIPLICATION.

WRITTEN EXERCISES.

264. *1.* Multiply .56 by .4.

OPERATION.

$$\begin{array}{r} .56 \\ .4 \\ \hline .224 \end{array}$$

ANALYSIS.—Multiply as in integers, and since *hundredths* multiplied by *tenths* produces *thousandths*, the product must contain *three* decimal places, or as many as both factors contain.

RULE.—*Multiply as in multiplication of integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.*

1. If there are not as many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

2. To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places toward the *right* as *there are ciphers* in the multiplier.

	(*2.*)	(*3.*)	(*4.*)	(*5.*)
Multiply	4.64	53.062	.1346	675.1
By	3.35	4.53	.203	.008

What is the product of

6. 6 times $2.45.
7. 9 times .326.
8. 26 times 4.007.
9. 15 times .0038.
10. .5 of $6.25.
11. .22 of 3.84.
12. .07 of $12.031.
13. .042 of .506.
14. $3.6 × .045.
15. .743 × 6.04.
16. 28.7 × .029.
17. .186 × $4.02.

18. Of 3000 pounds of wool, at 37½ cents?

19. In 1 rod is 16.5 feet; how many feet in 30.005 rods?

20. If 1 dollar in gold be worth 109⅛ cents in currency, what is $500 in gold worth?

DIVISION.

WRITTEN EXERCISES.

265. *1.* What is the quotient of .624 divided by .6 ?

OPERATION.

$$\begin{array}{r} 6\,)\,.6\,2\,4 \\ \hline 1.0\,4 \end{array}$$

ANALYSIS.—Divide as in integers, and since the dividend contains *three* decimal places and the divisor *one*, the quotient must have *two* decimal places.

RULE.—*Divide as in division of integers, and from the right of the quotient point off as many figures as the decimal places in the dividend exceed those in the divisor.*

1. If the number of figures in the quotient be *less* than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by *prefixing* ciphers.

2. The dividend must always coutain at least as many decimal places as the divisor, before commencing the division.

Divide

3. 8.176 by 7.

4. .675 by .15.

5. $26.64 by 8.

Divide

6. $41.42 by $5.45.

7. 3.76 by .8.

8. 17.6 by 44.

9. Divide 88.476 by 1.2; by 3.6 ; by 1.04 ; by .2½.*

10. Divide $56.05 by .59; $408.37½ by 27.

11. Divide $6.45 by $.45 ; $52 by $.65; 293.75 by 45¾.

Find the *price* of 1 bushel, 1 pound, etc.

12. If 125 bushels of oats cost $62.50 ?

13. If 35 pounds of sugar cost $2.80 ?

14. If 144 bushels of wheat cost $168.48 ?

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* Common fractions are reduced to decimals by annexing ciphers to the numerator, and dividing by the denominator ; thus, ½ = 1.0 + 2 = .5 ; ¼ = 1.00 ÷ 4 = .25 ; ⅜ = 3.000 ÷ 8 = .375.

Hence 2½ = 2.5 ; .5¼ = .525 ; 12¾ = 12.75 ; 7⅜ = 7.375, etc.

APPLICATION OF DECIMALS.

266. The most important computations involving *decimals* are those pertaining to *Percentage.*

267. ***Per Cent*** means *by the hundred,* and is often expressed by the *sign %.* It means *hundredths,* or a certain number of parts of every hundred parts.

Thus, 10 *per cent* of any number of apples, means 10 apples of every 100 apples; 5 *per cent* of every sum of money, means 5 cents of every 100 cents, or $5 of every $100. Therefore 5 *per cent* (5%) of *one* (1) equals *five hundredths* $= .05 = \frac{5}{100} = \frac{1}{20}$.

268. The ***Rate per Cent*** is the number of units for every hundred, or the number of hundredths for every unit, and is commonly expressed as a *decimal.*

Thus, 6% is 6 units for every *hundred* units, or 6 *hundredths* for every unit or *one;* in either case, written, .06.

5½% is written, .055; 6¼%, .0625; 7¾% .0775; 3⅛%, .03125, etc.

269. The ***Base*** is the number on which the percentage is computed.

Thus, in the expression 6% of $25, the *base* is $25.

270. The ***Percentage*** is the result obtained by taking a certain per cent of the base.

Thus, in the statement, 5% of $40 is $2, the rate *per cent* is .05, the *base* is $40, and the *percentage* is $2.

271. The *base* plus the *percentage* is the ***Amount;*** and the *base* less the *percentage* is the ***Difference.***

Thus, if the *base* is $40, the rate *per cent* .05, and the *percentage* $2, $40 + $2 = $42, is the *amount,* and $40 − $2 = $38 is the *difference.*

Hence, the Base is the *Multiplicand,* the Rate per cent is the *Multiplier,* and the Percentage is the *Product.*

272. In the applications of percentage the three principal elements are the Rate, the Base, and the Percentage, any two of which being given, the other may readily be found, since the Base and Rate are *factors* in the Percentage, as shown in the three following questions:

1. In an examination of 40 pupils, 5% of them failed. What percentage, or how many failed?

40 = *Base.*
.05 = *Rate.*
2.00 = *Percentage.*

FORMULA.—*Percentage* = *Base* × *Rate.*

2. A boy missed 2 questions in answering 40. What per cent did he miss?

Base. Percentage. Rate.
40)2.00(.05
200

FORMULA.—*Rate* = *Percentage* ÷ *Base.*

3. A man spent 2 years in teaching, which was 5% of his age when he died. What was his age?

Rate. Percentage. Base.
.05)2.00(40
20
0

FORMULA.—*Base* = *Percentage* ÷ *Rate.*

In like manner, solve the following, and name the elements of each:

4. What is 4% of $300? Of $175? Of $48.75? Of $96? Of $1275.60?

5. What is 7½% of 200 sheep? 9% of 150 miles? 8% of $250? 6¼% of 5000 bushels?

6. What is 25% of 212 pounds? 15% of 94 acres? 40% of 475 men? 33⅓% of 960 barrels?

7. What per cent of $30 is $6? Of 200 lb. are 50 lb.? Of 160 rd. are 48 rd.? Of 400 men are 80 men?

8. Of what number are 30 lb. 20%? $75, 5%? 56 doz., 7%? 54 miles, 18%?

9. A farmer having 180 sheep, sold 20% of them. How many did he sell? How many had he left?

10. A man bought 1200 barrels of apples, and lost 12½% of them by rot. How many barrels did he lose?

273. The preceding applications are applicable also to transactions in Commission, Profit and Loss, Discount, Premiums, etc.

1. What commission is paid for collecting $384 at 3½%?

2. A commission merchant sells goods to the amount of $5487.50. What is his commission at 2%?

3. Bought a horse for $150, and sold him at 15% profit. What was my gain, and what the selling price?

4. Bought 28 cords of wood at $3.50 a cord, and sold it so as to gain 33%. What did I make?

5. A commission merchant sells 400 barrels of potatoes at $2.25 a barrel, and 245 barrels of apples at $3.30 a barrel. What is his commission for selling, at 5%?

6. Find the premium for insuring a house and furniture, valued at $900, at 3¼%.

7. A merchant bought cloth at $4 a yard, and sold it at 25% advance. What was his profit, and the selling price?

8. A grocer bought sugar at 12 cents a pound. For what must he sell it to gain 25%?

9, Butter bought at 32 cents a pound, was sold at a loss of 12½%. At what price did it sell?

10. A debt of $375.25 due in a year, was discounted at 10% for cash. How much was the discount? How much cash was paid?

11. Imported books from England for which I paid $150. The duty was 15%; the freight $5.75. What did the books cost me?

12. A merchant paid $5460 for a stock of goods, which he sold at an advance of 25%. After deducting $416.50 expenses, what was his gain?

Find the *base*

13. If the percentage is 16, and the rate per cent 4.

14. If the percentage is 105, and the rate per cent 35.

15. If the percentage is $975, and the rate per cent 15.

16. Fifty pupils is 12½% of the whole number attending a certain school. What is the number?

17. A horse was sold at a profit of $40, which was 20% of what he cost. What did he cost?

Find the *rate per cent,*

18. If the percentage is 40, and the base 250.

19. If the percentage is $13.20 and the base $240.

20. A man gave $45 for the use of $900 one year; what rate per cent did he pay?

21. A carriage that cost $150 sold at a loss of 10%. What did it bring?

22. A grocer bought 1078 lb. of sugar, at 8½ cents a pound, and sold it at a profit of 18%. What did he gain?

23. What is the premium for insuring a cargo of 5000 bushels of wheat, valued at $1.50 a bushel, at 2½%?

INTEREST.

274. The operations of *Interest* are the same as the other operations of percentage, except that they involve the additional element of *time.* The *principal* is the *base,* the *interest* is the *percentage,* and the interest on one dollar for one year is the *rate per annum.*

275. ***Interest*** is a sum paid for the use of money.

276. ***The Principal*** is the sum for the use of which interest is paid.

277. ***The Rate of Interest*** is the per cent, or number of hundredths of the principal, paid for its use *one year.*

278. ***The Amount*** is the sum of the principal and the interest.

279. PRINCIPLE.—*The interest is the product of three factors, namely: the principal, rate per annum, and time.*

1. Find the interest on $75 for 2 years, at 6%.

OPERATION.

$75 = Principal or base.
.06 = Rate per annum, or rate.
$4.50 = Interest for 1 year.
2 = Time in years.
$9.00 = Interest for 2 years.

ANALYSIS.—Since the interest of any sum at 6% for 1 yr. is .06 of the principal, the interest of $75 at 6% for 1 yr. is .06 of $75, or $75 × .06 = $4.50; and the interest for 2 yr. is 2 times the interest for 1 yr., or $4.50 × 2 = $9.

2. What is the interest on $120.60 for 3 yr. 4 mo. at 8%?

OPERATION.

$120.60 = Principal.
.08 = Rate per cent.
12) $9.6480 = Int. for 1 year.
.804 = Int. for 1 month.
40 = Time in months.
$32.160 = Int. for 3 yr. 4 mo.

The interest for 1 year may either be multiplied by the time in years and fraction of a year ($3\frac{1}{3}$ yr.); or, the interest of 1 mo. may be multiplied by the time in months (40 mo).

RULE.—I. *Multiply the principal by the rate per cent, and the product will be the interest for 1 year.*

II. *Multiply the interest for 1 year by the time in years and the fraction of a year, and the product will be the required interest.*

In ordinary business transactions involving interest 30 days are considered 1 month, and 12 months, 1 year.

Find the interest of

3. $500 for 3 yr. at 4%.
4. $650 for 2 yr. at 5%.
5. $150 for 3 yr. at 6½%.
6. $740 for 4 yr. at 7%.
7. $225 for 8 mo. at 10%.
8. $325 for 2 yr. 6 mo. at 8%.
9. $465.50 for 3 yr. 4 mo. at 10%.
10. $95.75 for 4¼ yr. at 3¾%.
11. $84 60 for 1 yr. 9 mo. at 6%.
12. $1000 for 1 yr. 6 mo. at 12%.

Find the interest

13. Of $372 for 1 yr. 4 mo. at 6%; at 4%; at 8%; at 7%.
14. Of $896 for 9 mo. at 10%; at 9%; at 12%; at 7%; at 3%.
15. Of $175 for 1 yr. 7 mo. 15 da. at 7%.

OPERATION.

	$175	=Principal.
	.07	=Rate per cent.
2	$12.25	=Int. for 1 yr.
6	6.125	=Int. for 6 mo.
2	1.020	=Int. for 1 mo.
	.510	=Int. for 15 da.
	$19.905	=Int. for 1 yr. 7 mo. 15 da.
	$175	=Principal.
	$194.905	=Amount for 2 yr.

ANALYSIS.—First multiplying the principal by the rate per cent, we have $12.25, the interest for 1 year.

Then, for 6 mo. take ½ of 1 year's int., for 1 mo. take ⅙ of 6 months' int., and for 15 da. take ½ of 1 month's int. The sum of the several results is the entire interest for the whole time

Adding the principal to the interest gives the *amount.*

Find the interest

16. Of \$227.10 for 3 yr. at 7%. At 8%. At 5%.

17. Of \$1320 for 1 yr. 5 mo. at 6%. At 4½%. At 9%.

18. Of \$62.84 for 10 mo. 20 da. at 6%. At 12%. At 8%.

19. Of \$413.75 for 9 mo. 10 da. at 6%. At 7%. At 5%.

20. Of \$2162.94 for 1 yr. 8 mo. at 10%. At 6%. At 6¼%.

21. Of \$56.80 for 4 yr. 1 mo. at 4½%. At 3%. At 7½%.

An easy and uniform method of computing interest is to place the principal, the rate, and the time in months on the right of a *vertical* line, and 12 on the left; or if the time is short and contains days, reduce to days, and place 360, or its factors 12 and 30, on the left. After cancelling equal factors on both sides of the line, the product of the remaining factors on the right, divided by the factor, or product of the remaining factors, if any, on the left, will give the required interest.

22. What is the interest of \$184.80 for 1 yr. 5 mo. at 5%?

OPERATION.

$$\begin{array}{r|l} & \overset{\$15.40}{\$\cancel{184.80}} \\ \cancel{12} & .05 \\ & 17 \\ \hline & \$13.09\ \textit{Ans.} \end{array}$$

Analysis.—\$184.80 × .05 gives the interest for 1 yr. or 12 months, which divided by 12 gives the interest for 1 month; the quotient multiplied by 17, the number of months in 1 yr. 5 mo., gives \$13.09, the interest required.

23. What is the interest of \$240 for 2 mo. 18 da. at 7%?

OPERATION.

$$\begin{array}{r|l} & \$\cancel{240}^{6} \\ {}^{3}\cancel{12} & .07 \\ \cancel{30} & \cancel{78}^{26} \\ \hline 3 & \$10.92 \\ \cline{2-2} & \$3.64 \end{array}$$

Analysis.—\$240 × .07 gives the interest for 1 yr. or 360 days, which divided by 360, or its factors 12 and 30, gives the interest for 1 day; the quotient multiplied by 78, the number of days in 2 mo. 18 da., gives \$3.64, the required interest.

Find the interest at 6% of

24. $267.27 for 6 mo.
25. $146.18 for 1 mo. 21 da.
26. $256.84 for 2 yr. 4 mo.
27. $38.90 for 1 yr. 1 mo.
28. $146.48 for 6 mo. 10 da.
29. $275.50 for 4 mo. 24 da.

Find the interest

30. Of $276.12 for 1 yr. 6 mo. at 7%. At 5%. At 12%.

31. Of $600 at 6% for 2 yr. 8 mo. For 3 yr. 9 mo.

Find the interest and the amount,

32. Of $450 for 3 yr. 9 mo. at 6%. For 8 mo. at 7%.

33. Of $247 for 5 yr. 3 mo. at 5½%. For 10 mo. at 8%.

34. Of $500 for 4 yr. 2 mo. at 10%. For 11 mo. at 5%.

Find the amount,

35. Of $63.70 for 1 yr. 3 mo. 20 da. at 7%. At 8%.

36. Of $475.57 for 1 yr. 7 mo. at 6%. At 5½%.

37. Of $525.35 for 3 yr. 6 mo. at 7½%. At 9%.

38. Of $1140.63 for 9 mo. 15 da. at 10%. At 4½%.

39. If a person borrow $540.75 at 8%, how much will be due the lender at the end of 2 yr. 4 mo.?

40. Bought a house for $5400. Paid $2000 cash on delivery, $1500 in 9 mo. and the remainder in 1 yr. 4 mo. with 7% interest. What was the whole amount paid?

41. A man invested $1500 for 2 yr. 4 mo. 18 da. at 8%. What was the interest and the amount?

42. Paid a debt of $175.50 which had been upon interest 11 mo. 16 da. at 6%. What was the amount of the payment?

43. A note was given for $360, bearing interest at 10%, dated May 1st, 1877. What amount will be required to pay the note on the 20th of Sept. following?

EXERCISES IN PERCENTAGE.

Find the *percentage*, which is equivalent to the commission, or premium, or gain or loss, on each of the following numbers, as the *base*, at ½%, 1%, 2½%, ¾%, 4%, 6¼%, 5%, 8%, 10%, 12½%, 15%, 24%:

1. $575
2. $960
3. $1216
4. $4650
5. $125.50
6. $304.08
7. $75.90
8. $641.18
9. $1056.20
10. $3028.84
11. $736.16
12. $2046.00
13. $3009.65
14. $2780.50
15. $42872.48
16. $25014.80

Find the *interest* of each of the following principals, for the time specified, at 3%, 4%, 4½%, 5%, 6%, 7%, 8%, 10%.

17. $48.75, 2 yr. 6 mo.
18. $276.60, 1 yr. 9 mo.
19. $750.00, 4 yr. 4 mo.
20. $2400.00, 2 yr. 1 mo.
21. $156.00, 0 yr. 11 mo.
22. $376.40, 1 yr. 2 mo. 10 da.
23. $700.00, 2 yr. 3 mo. 15 da.
24. $94.80, 4 yr. 6 mo. 20 da.
25. $1000.00, 0 yr. 9 mo. 24 da.
26. $1250.60, 0 yr. 1 mo. 19 da.

Find the interest and the *amount*, at above rates, on each of the following:

27. $327.50, 1 yr. 6 mo.
28. $580.00, 2 yr. 3 mo.
29. $87.60, 4 yr. 9 mo.
30. $615.30, 2 yr. 4 mo. 15 da.
31. $1200.00, 0 yr. 6 mo. 12 da.
32. $750.80, 3 yr. 2 mo. 25 da.

Find *numbers*, of which the numbers in each column below are 2%, 3%, 4½%, 5%, 6%, 8½%, 10%, 12½%, 25%, 40%.

Also, find what *per cent* each number on the left in each double column is of the number at the right of it; as 9 of 12, 16 of 24, etc.

33. 9 12
34. 7 56
35. 30 40
36. 20 80
37. 16 24
38. 40 250
39. 54 360
40. 75 300
41. 18 150
42. $50 $450
43. $24 $300
44. 15 bu. 150 bu.

MISCELLANEOUS EXAMPLES.

1. How many bushels of oats worth 60 cents a bushel, will pay for ¾ of a barrel of flour at $8.40 a barrel?

2. What is the value of a piece of land 50 rods long and 40 rods wide, at $112.50 an acre?

3. If a bale of cotton weighs 500 pounds, what will be the value of 300 bales at 14½ cents a pound, after deducting 5% for freight, commission and storage?

4. Gave a note of $250 for 2 years; what was the amount at 6%?

5. What must be paid to enclose a farm ½ mile square, with a fence, at a cost of $.80 a rod?

6. A man bought 20 head of young cattle at $16.50 a head, and gave his note payable in 8 mo. at 8%. At the end of that time he sold the cattle at $18.75 a head. How much did he get for keeping them?

7. A grocer purchases 8 barrels of sugar, each weighing 2 cwt. 25 lb., at $7.50 per cwt., and retails it at 9½ cents a pound. What was his whole gain?

8. What is the 3d power of 8? The 4th power of 9? The product of the 2d power of 15 and the 3d power of 12?

9. A residence is rented for $57.50 per month. What is the rent for 3 yr. 6 mo.? If the tenant gets a discount of 5% for cash in advance, what is the rate per month? How much for the whole time?

10. How many rods of fence will inclose a section of land? A half-section? A quarter-section?

11. What is the solid contents of a cubical block of ice, whose edge is 18 inches?

11. If ¾ of a hundred pounds of sugar cost $7.125, how much can be bought for $49.875 ?

12. Bought 2500 bu. of wheat in Chicago, at $1.40 a bushel; paid 4½% insurance, 3% freight, and 2½% commission for selling; it was sold in New York at $1.90 a bushel. What was the whole gain ?

13. How many square feet of boards will make a box and cover, 14 ft. long, 6 ft. wide, and 4 ft. high ?

14. The distance from New York to Chicago is 961 miles ; from Chicago to Omaha, 492 miles; from Omaha to San Francisco is 1931 miles. A train of cars was run from New York to San Francisco in 84 hours. What was the average speed per hour ?

15. A bushel contains 2150.4 cubic inches. How many more cubic inches in a bushel than in a cubic foot ?

16. Bought a hhd. of sugar, containing 9 cwt. 44 lb., for $84.96; paid $4.72 for freight. At what price per pound must it be sold to gain 20% on the cost ?

17. If a township of land is equally divided among 288 families, how many acres does each receive ? What part of a section ?

18. How many years, months, and days from your birthday to this date; or, what is your age ?

19. How many days from June 20th to Jan. 10th ?

20. A planter has 600 acres of cotton, from which 6 acres are selected to determine the average yield and to estimate the entire crop. The first acre produced 1½ bales, the second 2¼ bales, the third 1¼ bales, the fourth 2⅛ bales, the fifth ⅞ of a bale, and the sixth 2½ bales. At how many bales shall he estimate his crop ?

21. On a bill of $285, a merchant discounts 5 per cent for cash. How much does he receive?

22. An agent sold my house and lot for $6525. What was his commission at 2 per cent?

23. When gas is $2.50 per 1000 feet, if a discount of 20% is made for cash, what is the price? What is the cost of 3500 feet?

24. The earth's circumference at the equator is 25000 miles. How many days would it take a locomotive to travel that distance at the rate of 25 miles an hour?

25. A farmer sold 125 bu. of corn at $.62½ a bushel, and 2½ tons of hay at $18½ a ton. He received in payment 96 lb. of sugar at 8⅓ cents a ponnd, 28 lb. of tea at $⅝ a pound, 3 bbl. of flour at $7.87½, and the remainder in cash. What amount of cash did he receive?

26. How many acres in a farm 120 rods square?

27. Paid 7 cents a pound for 2480 pounds of pork, and afterward lost 10 per cent on the cost, in selling it. What was my whole loss?

28. Sent to my agent in Chicago $2741.25, for which he purchased 475 barrels of flour; what was the price per barrel, and what his commission at 2½%?

29. A merchant buys 5500 bushels of wheat at $1.58 a bushel, ships it to New Orleans, paying 4 cents a bushel for freight, and 2% for insurance and commission. The wheat is sold at $1.76 a bushel. What does he gain on the investment?

30. How many square feet of surface in the walls of a house 30 ft. long, 24 ft. wide, and 16 ft. high? How many square feet in the floor and ceiling of this house?

31. What is the cost of 5 tons of wheat, at $1.50 a bushel?

32. Required the interest and the amount of $1500 from May 1 to Sept. 20, at 7%.

33. If I buy 225 barrels of flour for $1125, and sell the same for $1800, what do I gain on each barrel?

34. A commission merchant sells 400 barrels of potatoes at $2.25 a barrel, and 345 barrels of apples at $3.20 a barrel. What is his commission for selling, at 5 per cent?

35. If a man has 24⅞ bushels of clover seed, and he sells ¾ of it, how much has he left?

36. How many cords of wood in a pile 38 ft. long, 8 ft. wide, and 6 ft. high; and what is its value at $3½ a cord?

37. An Illinois farmer received $.45 a bushel for 4 loads of corn; the first contained 48 bu. 28 lb., the second 2626 lb., the third 1898 lb., and the fourth 41 bu. 50 lb. What did he receive for the whole?

38. Paid $1.24 for 15.5 pounds of beef. How much was the price per pound?

39. Bought a building lot, 50 ft. front and 120 ft. deep, at $5 per front foot. What did it cost? What would the same lot cost at $.50 per square foot?

40. If 3 barrels apples cost $19.125, what will be the cost of 100 barrels?

41. Find the interest on $78.75 for 1 yr. 9 mo., at 4%.

42. A merchant bought 350 bushels of wheat at $1.34 a bushel, and sold the flour at 16% advance on the cost of the wheat; what was his gain?

43. Bought 125 tons of coal at $6⅝ a ton, and sold it at a loss of 12½%; what was the whole loss?

44. Find the amount of $1625 for 1 yr. 6 mo., at 8%.

45. How many cubic yards of earth must be thrown from a cellar 40 ft. long, 30 ft. wide, 6 ft. deep; and what is the cost of the excavation, at 12½ cents a cubic yard ?

46. If 6 pounds of cheese cost $⅘, what will 10 pounds cost?

47. How much wheat, at $1.25 a bushel, must be given for 50 bushels of corn at $.70 a bushel ?

48. A grocer bought 12 firkins of butter, each containing 56 pounds, at 24 cents a pound; he afterward sold 5 firkins, at 30 cents, and 7 firkins at 28 cents a pound. What was his whole gain ?

49. At $4.25 per bushel, how much clover-seed can be bought for $11.6875 ?

50. What will be the cost of 3 hogsheads of sugar, each weighing 15 cwt., at 8 cents a pound?

51. A miller sold 256 barrels of flour, at $6.80 a barrel, which was $475.60 more than the wheat from which it was made, cost him. What was the cost of the wheat?

52. A farmer put all his grain into 4 bins: in the first he put ⅜ of it, in the second ¼, in the third ⅙, and in the fourth 40 bushels. How many bushels of grain had he?

53. A farmer took 3 tons of hay to market, for which he received $12.62½ a ton. He bought 2 barrels of flour at $9¼ a barrel, 25 pounds of coffee at $.37½ cents a pound, and 3 hand-rakes at $.75 each. How much money had he left?

54. A vessel loaded with 4500 bushels of corn, was overtaken by a storm at sea, and it was found necessary to throw overboard 25% of her cargo. What was the whole loss, at 60 cents a bushel?

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